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Centrality Analysis of a Weighted Iranian Provinces Adjacency Network

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ABSTRACT

Recently, network theory has gained prominence for modeling and analyzing a wide range of phenomena to achieve a deeper understanding. In this paper, an undirected weighted network is employed to model the adjacency structure of the provinces of Iran. Each province is represented by a node in the network, and whenever two provinces share a boundary, a weighted edge is established between them. The edge weight is determined by the distance between the centers of the two adjacent provinces and lies in the interval [0,1]; closer provincial centers yield weights nearer to 1. The resulting network, hereafter termed the Weighted Iranian Provinces Adjacency Network (WIPAN), is analyzed to extract key network statistics. However, the core focus of the study involves the computation of various centrality measures on this network. For this analysis, twelve well-established centrality metrics-widely utilized in network science literature-were selected. This suite encompasses both fundamental measures, such as degree, closeness, and betweenness centrality, and more advanced or hybrid methods, including Katz, eigenvector, PageRank, and Laplacian centrality. Each method, viewed from its own standpoint, answers which nodes are most central in the network. Based on a consensus ranking of the centrality measures, node 10 emerges as the most central node in the network, while node 22 is the least central. The findings of this study establish a foundational basis for further analyses and can be applied to a broad range of planning problems as well as to decision-making at both micro and macro levels.

1 Introduction

Complex systems consist of multiple interacting components whose relationships can be effectively modeled using network theory. In such models, components are represented as nodes (vertices), and their interactions as edges (links). Depending on the network under study, these nodes and edges may carry different meanings. Consequently, networks have been widely adopted to model diverse complex systems, including collaboration networks, infrastructure systems, social networks, and biological networks, among others.

This modeling approach offers several advantages. Abstracting a complex problem into a network representation simplifies understanding and facilitates access to analytical solutions. Furthermore, leveraging estab-

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lished concepts and algorithms from network theory enables researchers to extract valuable information from the system and generate meaningful predictions about its future behavior.

Among the various discussions in network analysis-ranging from clustering to connectivity and community detection the concept of centrality measures occupies a particularly important position. These measures allow researchers to move beyond simple topological descriptions and quantify the relative influence of individual nodes within a system. By identifying which nodes act as key bridges, dominant hubs, or peripheral elements, centrality analysis provides crucial insights for strategic decision-making regarding the flow of information, resources, or influence across a network. There are many applications of centrality measures in sociology, psychology, economics, anthropology, biology, terrorism, traffic, neural, business, etc [1].

In [2], a weighted network for intercity freeway traffic in China was constructed. Subsequently, by incorporating socio-economic factors into this spatial network and applying an improved centrality method to it, predictions of highway traffic volumes were made. In [3], a composite network integrating China's air and high-speed rail networks was formed in the form of a directed weighted network, and network analysis algorithms and centrality measures were applied to yield notable results. Similarly, [4] presents the construction and analysis of a protein-protein interaction (PPI) network, where centrality measures such as degree, betweenness, closeness, and clustering coefficient are applied to identify 20 influential proteins, including hubs (high connectivity) and bottlenecks (high betweenness centrality). These proteins occupy strategic positions within the PPI network. The findings offer important insights into cancer biology and hold substantial promise for personalized medicine, drug design, and the development of novel cancer therapies. Understanding the role of these proteins in cancer can also inform the development of targeted therapies and new anticancer drugs.

In [5], a correlation network among 86 types of assets (including stock indices, bond indices, foreign exchange rates, commodity futures, and cryptocurrencies) was created, and then centrality-based methods were employed to analyse the selection of assets for constructing an optimal portfolio. In [6], a network-based model of interprovincial information communications in China is examined to reveal how information spreads between provinces. This study analyses the spatial distribution and connectivity of information networks to illuminate key nodes, regional differences, and topological evolutions over time. The research explains how information dissemination at the provincial level accelerates or stalls and identifies the drivers shaping information connections. Similarly, [7] employs network analysis to study the network of China's provincial economies and uses network-analytic methods to examine the pathways of communication, interdependencies, and the structure of economic exchanges among China's provinces. The findings indicate which provinces act as central components and vital links in China's production and trade. Focusing on network measures such as degree, centrality, and qualitative clustering, this study explains which economic, geographic, and policy factors influence the distribution of resources, the flow of goods, and interprovincial relationships.

In the context of climate change adaptation, [8] offers a comprehensive review of centrality measures for transportation networks. The paper notes that the transportation sector-a fundamental pillar of society and the economy-will be negatively affected by climate change, particularly through extreme weather events that may render critical nodes inoperable. By identifying these critical nodes using centrality measures, resources can be allocated to protect them and ensure their functionality during disasters. Assessing 17 centrality measures, the study argues that understanding and safeguarding key network elements can enhance resilience and reduce vulnerability.

Among various topics in social network analysis, centrality measures play a key role by quantifying node importance and are directly linked to identifying opinion leaders. As discussed in [9], these measures help determine which nodes can maximize information diffusion, making them essential for understanding how ideas and influence spread through a network. Centrality metrics thus serve as a bridge between structural network properties and the strategic challenge of influence maximization, particularly in contexts such as viral marketing or social contagion.

As the preceding review demonstrates, the principal challenge in network analysis lies in faithfully modeling real-world systems using network concepts and translating the methodology into practical solutions. Addressing this challenge, this study applies network analysis to model Iran's provincial adjacency structure as a weighted, undirected network. Here, nodes represent the 31 provinces, and edges represent their 74 shared borders. Each edge carries a weight (from 0 to 1) based on the geographic proximity between provincial centers, with the closest pair (e.g., Tehran-Karaj) assigned a weight of 1 and the most distant pair assigned 0.

The NetworkX package in Python was employed to calculate twelve established centrality measures, encompassing both local and global metrics. These metrics define "centrality" in distinct ways-such as by direct connections (degree), average proximity (closeness), control over network flows (betweenness), or connections to influential neighbors (eigenvector).

The core of the research investigates the practical implications of these centrality profiles. For national planning frameworks, the results yield actionable insights: provinces central in closeness are prime candidates for emergency hubs; those high in betweenness are suited for operational command; and those with high degree can serve as logistical terminals. Identifying less central nodes also holds strategic value. Consequently, these findings establish a rigorous, data-driven foundation for strategic planning. The framework's relevance is compellingly validated by its direct applicability to politically and logistically complex questions, such as the relocation of the national capital.

The main contributions of this paper are threefold. First, the Weighted Iranian Provinces Adjacency Network (WIPAN) is introduced as a novel framework for analyzing provincial adjacency networks using an integrated set of twelve established centrality measures. Second, by applying this framework to Iran's provincial network, empirical insights into regional connectivity patterns are provided, and practical implications for infrastructure and emergency planning are offered. Third, a robust methodology is offered through the consensus-based ranking approach, which can be generalized to other geographical networks, providing both methodological advancements and practical decision-support tools.

This paper is structured as follows: Section 2 introduces the Weighted Iranian Provinces Adjacency Network (WIPAN) and presents a set of fundamental network measures computed for this model. Section 3 outlines widely used methodologies for calculating centrality in both weighted and unweighted networks. Section 4 then applies these centrality measures to WIPAN and reports the corresponding results. Finally, Section 5 provides conclusions and suggests potential directions for future research.

2 The Iranian Provinces Adjacency Network and Its Specifications:

Iran, as one of the world's expansive countries, comprises 31 provinces, each including a central capital and several cities. It is natural to assume that awareness of the capabilities, connections, and strategic importance of the provinces can influence the design of national-level strategies. Any analysis or review that can support decision-making in this regard may prove useful. As previously mentioned, one of the methods for analyzing a complex system composed of multiple interdependent components is to model it as a network of elements and the interactions between them. Here, this approach has been employed to model the adjacency structure of Iran's provinces. Naturally, based on geographical position, every province shares borders with several other provinces and is considered adjacent to them. To construct the adjacency network of Iran's provinces, each province is represented as a node in the modeled network, and the existence of a shared border between two provinces is represented as an edge between their corresponding nodes. Through examination of the geographic borders among the provinces, 74 adjacency relations have been observed; accordingly, the adjacency network of Iran's provinces contains 31 nodes and 74 edges. The network, designated as the Weighted Iranian Provinces Adjacency Network (WIPAN), is shown in Figure 1. In the presentation of this network, each of Iran's provinces is assigned a number in the range from 0 to 30, and an edge is drawn between the nodes corresponding to two provinces if a shared border exists. Given the undirected nature of adjacency, the adjacency network is an undirected network. The placement of the WIPAN nodes in Figure 1 corresponds to the geographic location of the center of the respective province. The node number assigned to each province is provided in Table 1. Information of various kinds can be embedded on the nodes and edges of this network to describe each province and its relationships. Edge information, typically referred to as edge weights, captures the strength of the relationship between the two end nodes. In WIPAN, edge weights represent the relative proximity of neighboring province centers and are numeric values in the interval [0, 1]. To compute the edge weights, the normalized distance between two adjacent nodes is first calculated (Equation 1). The edge weights for that pair of adjacent nodes are then obtained from this distance using Equation 2.

$$\text{normalized\_dist}_{(u,v)} = \frac{\text{dist}_{(u,v)} - \text{dist}_{\text{min}}}{\text{dist}_{\text{max}} - \text{dist}_{\text{min}}} \tag{1}$$

$$w_{(u,v)} = 1 - \text{normalized\_dist}_{(u,v)} \tag{2}$$

In this context, $dist_{max}$ and $dist_{min}$ denote, respectively, the greatest and smallest distances between provincial centers, while $dist_{(u,v)}$ denotes the distance between the centers of the two provinces whose proximity weight is being computed. The greatest distance between provincial centers ($dist_{max}$) occurs between nodes 20 and 29 (Semnan and Birjand) with a magnitude of 914 km. The smallest distance between provincial centers ($dist_{min}$) occurs between nodes 23 and 30 (Tehran and Karaj) with a magnitude of 43 km. Consequently, based on Equation 1, the normalized distance between nodes 23 and 30 is 0, and the normalized distance between nodes 20 and 29 is 1. For the remaining adjacent provinces, the weight lies in (0, 1) in accordance with $dist_{(u,v)}$. By employing the normalized distance, edge weights can be readily derived. Figure 2 presents the weighted adjacency network of Iran's provinces.

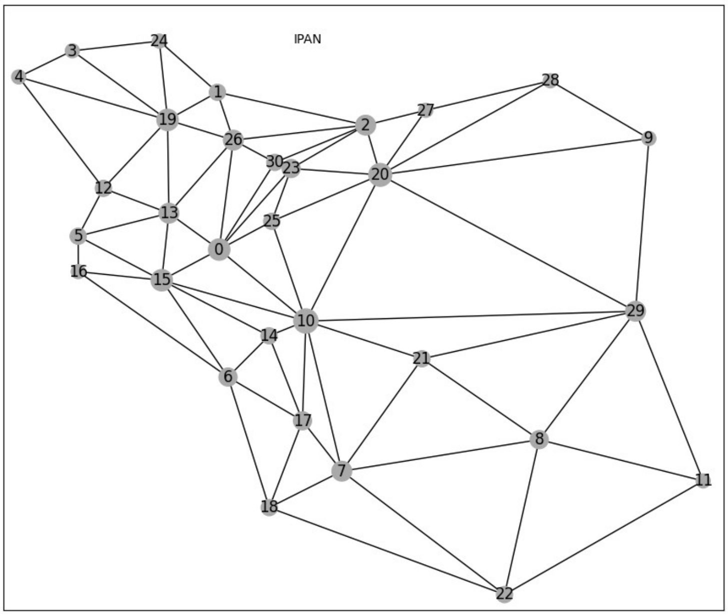


Figure 1. The Weighted Iranian Provinces Adjacency Network (WIPAN) with geographic node placement.

Network measures are numerical indices used to evaluate the characteristics of a network. Each of these measures reflects the reasons behind certain behaviors observed in a network. Numerous measures exist for assessing the structural properties of networks. In this section, some of the more important network measures are introduced, and their values are shown for WIPAN.

The simplest global measures of any network are the number of nodes and the number of edges, usually denoted by n and m. In WIPAN, we have $n = 31$ and $m = 74$. Each node v has a set of neighboring nodes connected to v by a direct edge. These nodes are called the neighbors of v or the adjacent nodes of v, and we denote them by $N(v)$. The degree of node v is the number of elements in the set $N(v)$; in other words, the degree of a node is the number of its neighbors. For a weighted network, the degree of a node is the sum of the weights of the edges incident to it [10]. It is computed as below:

$$\mathrm{dw}_v = \sum_{u \in N(v)} w_{(v,u)} \tag{3}$$

Many properties and behaviors of a node in a network are influenced by the node's degree. Likewise, the overall behavior of the network is also influenced by the degrees of the network's nodes. In fact, analyzing how the node degrees are distributed in a network explains many of its behaviors. In WIPAN, node 10 has the largest number of neighbors (i.e., $N(10) = 9$). When edge weights are incorporated, its weighted degree is 5.273. Nonetheless, the highest weighted degree in WIPAN belongs to node 0, which, despite having 7 neighbors, has a weighted degree of 5.396. The higher weighted degree of node 0 arises from the stronger (i.e., larger) weights of its connections to its neighbors, compared to node 10. The degrees (weighted) of the WIPAN nodes can be observed in Table 1 (dw_v). The average node degree (weighted) of this network is 3.15. In an undirected network with n nodes, the maximum number of edges is $n(n-1)/2$. In this case, all possible edges exist, and the network is complete. In most networks, many of those edges do not exist, so the network is not complete. Network density is the ratio of the number of edges present in the network to the total number of possible edges; the more edges a network has, the denser it becomes. The density of a network is obtained from Equation 4 [10]. Given that in WIPAN $n = 31$ and $m = 74$, the network density is 0.1591.

$$\text{density} = \frac{2m}{n(n-1)} \tag{4}$$

The shortest path between two connected nodes v and u in a weighted network with nonnegative weights (as is common in many network contexts) is the path that minimizes the total cost, i.e., the sum of the edge weights along the path. The shortest-path distance between all pairs of nodes in the network can be computed and stored in a matrix, commonly referred to as the distance matrix or all-pairs shortest-path

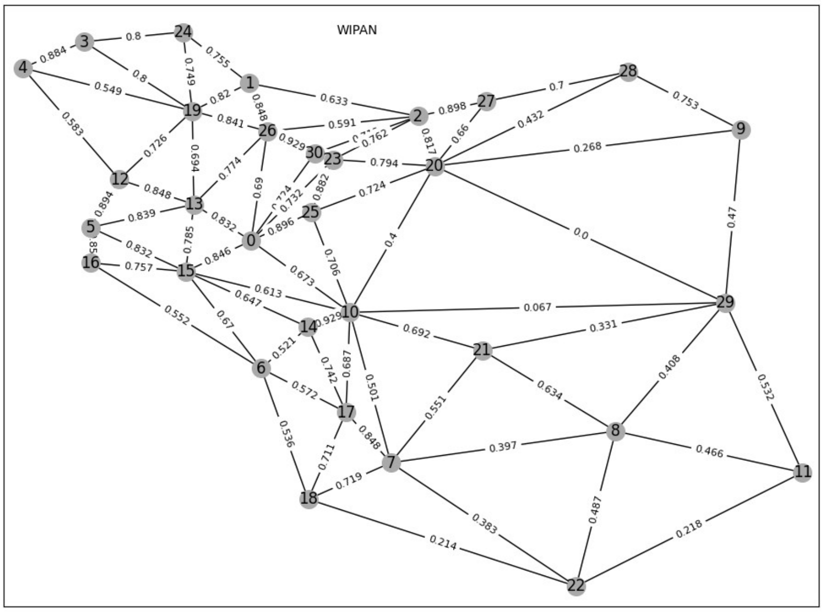


Figure 2. The Weighted Iranian Provinces Adjacency Network (WIPAN). Edge weights represent inter-capital proximity.

matrix. In this paper, it is termed the network distance matrix. The WIPAN distance matrix is shown in Figure 3.

Several measures can be computed from this matrix, among which eccentricity is one. The eccentricity of a node v is defined as the maximum distance from v to any other node in the network. In other words, the eccentricity of node v is the largest number in the row corresponding to node v in the distance matrix. For example, in WIPAN, the eccentricity of node 3 is 2131(km), since the largest entry in the row for node 3 is 2131. The values of node eccentricities in WIPAN are shown in Table 1 (E_v). Additionally, using the distance matrix, the important measure of the average distance from node v to the other nodes can be computed as well; the result of this calculation is also presented in Table 1 (A_v).

Using eccentricity, the network diameter and radius are defined as follows. The diameter of a network is the maximum eccentricity among its nodes, and the radius is the minimum eccentricity among its nodes [10]. Thus, referring to Figure 3, the maximum eccentricity of the WIPAN nodes is 2275 km and the minimum eccentricity is 1245 km; consequently, the diameter and radius of WIPAN are 2275 and 1245, respectively. In some sources, nodes whose eccentricity equals the network radius are defined as the network center. Based on this definition, in WIPAN, node 10, which has the minimum eccentricity, would be the network center (Isfahan Province).

The Wiener index, or Wiener number, is another global network measure defined as one half of the sum of all entries in the distance matrix. If we denote the sum of all entries in a row of the distance matrix as the distance sum (i.e., Equation 5).

$$s_v = \sum_{u \in V(G)} \mathrm{dist}_{(v,u)} \tag{5}$$

Then the Wiener index of the network is equal to one half of the sum of the s_v values over all nodes of the network, as given by Equation 6 [2]. It is straightforward to observe that the Wiener index (Wiener number) of WIPAN equals 406581.

$$W_G = \frac{1}{2} \sum_{v \in V(G)} s_v \tag{6}$$

The average path length (APL) is one of the most important measures of a network and plays a crucial role in evaluating the network's efficiency. The APL can be easily computed from the distance matrix. Additionally, it can be obtained with the Wiener index via Equation 7 [2]. Given that the WIPAN network has $n = 31$ nodes and using Wiener's index, the APL in this network is 874.367. It means the average distance between the provincial centers is 874.367 kilometers.

$$\mathrm{APL}_G = \frac{2W_G}{n(n-1)} \tag{7}$$

The clustering coefficient of a node is one of the most

0	488	526	711	770	366	507	804	999	1096	327	1507	364	189	431	177	431	642	936	494	416	638	1384	276	744	133	313	657	953	1182	283
488	0	362	416	560	597	974	1243	1438	1055	766	1908	480	414	870	644	765	1081	1375	199	544	1077	1823	322	256	467	175	493	797	1458	279
526	362	0	778	922	819	1033	1171	1366	693	694	1566	811	636	798	703	957	1009	1303	561	202	1005	1751	250	618	395	397	131	435	1116	293
711	416	778	0	144	633	1086	1466	1661	1447	989	2131	498	526	1093	756	801	1304	1533	217	767	1300	2046	545	217	690	398	909	1213	1681	502
770	560	922	144	0	541	1060	1522	1769	1591	1097	2275	406	581	1080	730	709	1347	1507	361	911	1408	2102	689	361	834	542	1053	1357	1825	646
366	597	819	633	541	0	519	981	1241	1462	569	1749	135	183	539	189	168	806	966	416	782	880	1561	569	677	499	422	950	1254	1424	526
507	974	1033	1086	1060	519	0	590	1158	1603	564	1666	654	560	460	330	433	415	447	869	923	875	1170	783	1130	640	799	1164	1460	1419	790
804	1243	1171	1466	1522	981	590	0	568	1563	477	1076	1116	993	442	792	1023	175	287	1249	1042	434	580	921	1499	776	1068	1302	1579	1059	964
999	1438	1366	1661	1769	1241	1158	568	0	1062	672	508	1363	1188	776	1052	1306	743	855	1444	1237	361	489	1116	1694	971	1263	1497	1320	558	1159
1096	1055	693	1447	1591	1462	1603	1563	1062	0	1245	954	1460	1285	1349	1273	1527	1560	1850	1230	680	1129	1551	902	1311	963	1049	562	258	504	945
327	766	694	989	1097	569	564	477	672	1245	0	1180	691	516	104	380	634	315	609	772	565	311	1057	444	1022	299	591	825	1102	855	487
1507	1908	1566	2131	2275	1749	1666	1076	508	954	1180	0	1871	1696	1284	1560	1814	1251	1363	1914	1364	869	724	1586	2164	1479	1733	1516	1212	450	1629
364	480	811	498	406	135	654	1116	1363	1460	691	1871	0	175	674	324	303	941	1101	281	780	1002	1696	561	542	497	414	942	1246	1546	518
189	414	636	526	581	183	560	993	1188	1285	516	1696	175	0	580	230	351	831	1007	309	605	827	1573	386	570	322	239	767	1071	1371	343
431	870	798	1093	1080	539	460	442	776	1349	104	1284	674	580	0	350	604	267	561	876	669	415	1022	548	1126	403	695	929	1206	959	591
177	644	703	756	730	189	330	792	1052	1273	380	1560	324	230	350	0	254	617	777	539	593	691	1372	453	800	310	469	834	1130	1235	460
431	765	957	801	709	168	433	1023	1306	1527	634	1814	303	351	604	254	0	848	880	584	847	945	1603	707	845	564	590	1088	1384	1489	694
642	1081	1009	1304	1347	806	415	175	743	1560	315	1251	941	831	267	617	848	0	294	1087	880	609	755	759	1337	614	906	1140	1417	1170	802
936	1375	1303	1533	1507	966	447	287	855	1850	609	1363	1101	1007	561	777	880	294	0	1316	1174	721	727	1053	1577	908	1200	1434	1711	1346	1096
494	199	561	217	361	416	869	1249	1444	1230	772	1914	281	309	876	539	584	1087	1316	0	550	1083	1829	328	261	473	181	692	996	1464	285
416	544	202	767	911	782	923	1042	1237	680	565	1364	780	605	669	593	847	880	1174	550	0	876	1622	222	800	283	369	333	537	914	265
638	1077	1005	1300	1408	880	875	434	361	1129	311	869	1002	827	415	691	945	609	721	1083	876	0	850	755	1333	610	902	1136	1387	625	798
1384	1823	1751	2046	2102	1561	1170	580	489	1551	1057	724	1696	1573	1022	1372	1603	755	727	1829	1622	850	0	1501	2079	1356	1648	1882	1809	1047	1544
276	322	250	545	689	569	783	921	1116	902	444	1586	561	386	548	453	707	759	1053	328	222	755	1501	0	578	145	147	381	685	1136	43
744	256	618	217	361	677	1130	1499	1694	1311	1022	2164	542	570	1126	800	845	1337	1577	261	800	1333	2079	578	0	723	431	749	1053	1714	535
133	467	395	690	834	499	640	776	971	963	299	1479	497	322	403	310	564	614	908	473	283	610	1356	145	723	0	292	526	820	1154	188
313	175	397	398	542	422	799	1068	1263	1049	591	1733	414	239	695	469	590	906	1200	181	369	902	1648	147	431	292	0	528	832	1283	104
657	493	131	909	1053	950	1164	1302	1497	562	825	1516	942	767	929	834	1088	1140	1434	692	333	1136	1882	381	749	526	528	0	304	1066	424
953	797	435	1213	1357	1254	1460	1579	1320	258	1102	1212	1246	1071	1206	1130	1384	1417	1711	996	537	1387	1809	685	1053	820	832	304	0	762	728
1182	1458	1116	1681	1825	1424	1419	1059	558	504	855	450	1546	1371	959	1235	1489	1170	1346	1464	914	625	1047	1136	1714	1154	1283	1066	762	0	1179
283	279	293	502	646	526	790	964	1159	945	487	1629	518	343	591	460	694	802	1096	285	265	798	1544	43	535	188	104	424	728	1179	0

Figure 3. All-pairs Shortest-path Distance Matrix of the WIPAN.

influential measures for understanding its local structure. It captures the tendency of a node's neighbors to be connected to each other. For a node v, the clustering coefficient is defined as the ratio of the number of triangles that include v to the number of length-2 paths centered at v. The average clustering coefficient of all nodes in the network is called the clustering coefficient of the network [2]. This measure, alongside other measures, is among the most important indicators for evaluating the behavior of a network. The clustering coefficient of node v is obtained from Equation 8.

$$C_v = \frac{\text{number of triangles that include } v}{\text{number of connected triples centered at } v} \tag{8}$$

In weighted networks, the strength of the relationship between two nodes should be considered when measuring the tendency of nodes to form triangles [11]. In other words, it is not only the existence of a connection that matters, but also the significance or intensity of the connections. Consider the edge weights $w_{u,v}$, the weighted clustering coefficient of node v computed as below:

$$\mathrm{Cw}_v = \frac{1}{\mathrm{dw}_v(d_v - 1)} \sum_{u,t \subseteq N(v)} \frac{(w_{u,v} + w_{v,t})}{2} a_{u,v} a_{v,t} a_{u,t} \tag{9}$$

Where $N(v)$ is the set of neighbors of v, d_v is the number of nodes in $N(v)$, dw_v is the weighted degree of v computed by Equation 3, $w_{u,v}$ is the weight of the edge (u, v), and $a_{i,j}$ is the adjacency indicator for the edge (i, j) (0 or 1). In this equation, when the surrounding edges are stronger, triangles through v contribute more to Cw_v. In other words, the measure blends topology (the presence of triangles) with edge strength (weights). The weighted clustering coefficients of the WIPAN nodes are presented in Table 1 (Cw_v). Based on the information in this table, the mean clustering coefficient of the nodes in WIPAN is 0.324.

Another important index for evaluating a network is its transitivity. Transitivity is closely related to the number of triangles in the network, and a single value can summarize the overall tendency of nodes to form triangles. Specifically, this measure indicates the fraction of three-node relations that participate in a triangle among those three nodes [2]. For example, in a friendship network, if a and b are friends and b and c are friends as well, what is the probability that a and c are also friends? The transitivity of a network can be derived from Equation 10.

$$T_G = \frac{3 \times \text{number of triangles}}{\text{number of connected triples}} \tag{10}$$

The total number of triangles in WIPAN is 44, and the total number of paths of length 2 is 320. Accordingly, using Equation 10, the transitivity of the network is 0.412. Transitivity is also referred to as the global clustering coefficient.

If one starts from a vertex v and, by traversing the edges of the network in various ways, returns to the vertex v, a cycle is formed. The length of the cycle is the number of edges along the traversed path. It is possible to return to the initial vertex by following paths of different lengths, thereby generating cycles of various lengths. The existence of more cycles in a network implies that data flow can be realized more smoothly within that network. The Estrada index is computed based on the counts of cycles of different lengths present in a network, by summing the weighted

Table 1. Key Topological Features and Provincial Identifiers for the WIPAN Nodes.

No	Province	dw_v	Cw_v	$E_v(km)$	$A_v(km)$
0	Markazi	5.396	0.260	1507	624
1	Gillan	3.059	0.382	1908	777
2	Mazandaran	4.417	0.254	1751	776
3	E. Azarbaijan	2.485	0.504	2131	948
4	W. Azarbaijan	2.017	0.448	2275	1023
5	Kermanshah	3.423	0.416	1749	747
6	Khuzestan	2.853	0.247	1666	869
7	Fars	3.402	0.222	1579	958
8	Kerman	2.395	0.221	1769	1094
9	Razavi Khorasan	1.493	0.148	1850	1171
10	Isfahan	5.273	0.140	1245	671
11	Sistan ...	1.217	0.278	2275	1466
12	Kordestan	3.053	0.372	1871	779
13	Hamedan	4.775	0.319	1696	677
14	Chaharmahal ...	2.842	0.452	1349	723
15	Lorestan	5.153	0.245	1560	667
16	Ilam	2.166	0.490	1814	838
17	Kohkiluye ...	3.564	0.341	1560	864
18	Bushehr	2.183	0.292	1850	1063
19	Zanjan	5.184	0.251	1914	762
20	Semnan	4.099	0.143	1622	725
21	Yazd	2.210	0.298	1408	861
22	Hormozgan	1.304	0.196	2102	1405
23	Tehran	4.172	0.405	1586	626
24	Ardebil	2.305	0.519	2164	958
25	Qom	3.210	0.496	1479	611
26	Qazvin	4.676	0.304	1733	666
27	Golestan	2.259	0.457	1882	873
28	N. Khorasan	1.886	0.343	1809	1067
29	S. Khorasan	1.812	0.077	1825	1166
30	Alborz	3.367	0.522	1629	636

counts of cycles of each length as the cycle length tends to infinity (i.e., considering the limit of infinitely long cycles). Shorter cycles are given larger weights, and longer cycles are given smaller weights. For this purpose, the weight of a cycle of length k is taken to be $1/k!$. Consequently, shorter cycles have a substantially greater influence on the Estrada index, while longer cycles have a negligible impact. The Estrada index is obtained from Equation 11, where k denotes the cycle length and M_k the number of cycles of length k [2]. The Estrada index for WIPAN is 431.391.

$$\mathrm{EE}_G = \sum_{k=0}^{\infty} \frac{M_k}{k!} \tag{11}$$

A planar network is one that can be drawn on a piece of paper without any of its edges crossing. WIPAN is a planar network with 31 vertices and 74 edges, and its chromatic number is 4. The chromatic number of a network is the minimum number of colours required to colour the vertices such that no two adjacent vertices share the same colour.

A subset of vertices of an undirected network, in which every pair of vertices is connected, is called a clique. The clique number of a network is the size of its largest clique [2]. The clique number of WIPAN is 3, since the largest clique present is a triangle. As noted earlier, the number of triangles in WIPAN is 44. The triangle with the greatest weight in WIPAN has weight 2.582 and connects the provincial capitals of Kermanshah, Hamedan, and Kordestan; the triangle with the smallest weight has weight 0.468 and connects the capitals of Isfahan, Semnan, and South Khorasan.

The efficiency of a pair of nodes in a network is the multiplicative inverse of the shortest path distance between the nodes, provided that a path exists; if no path exists, the efficiency of that pair is defined as zero. The average global efficiency of a network is the average efficiency of all pairs of nodes [12]. Accordingly, the average global efficiency of WIPAN is 0.460. The local efficiency of a node is the average global efficiency of the subgraph induced by the node's neighbors. The average local efficiency of a network is the average of the local efficiencies of its nodes [12]. Consequently, the average local efficiency of WIPAN is 0.728.

Assortativity measures the tendency of nodes to connect to other nodes that are similar (or dissimilar) with respect to a given property, such as degree, attribute, or type. The degree assortativity coefficient specifically describes the tendency of nodes to connect to other nodes with similar degrees. It quantifies how likely high-degree nodes are to connect to other high-degree nodes, and likewise for low-degree nodes. If high-degree nodes tend to connect to other high-degree nodes and low-degree nodes tend to connect to other low-degree nodes, the network is said to be assortative. Not all networks are assortative; for example, in social networks, low-degree nodes often connect to high-degree nodes, a pattern referred to as disassortativity. The assortativity coefficient ranges from -1 to 1. Values close to 1 indicate strong assortativity, while values close to -1 indicate strong disassortativity [13]. Mathematically, the degree assortativity coefficient is the Pearson correlation coefficient between the degrees of pairs of nodes at either end of edges. In weighted networks, a weighted Pearson correlation between the degrees k_i and k_j of the two nodes incident to each edge (i, j) is used, with edge weights $w_{i,j}$ serving as weighting factors. In WIPAN, the network assortativity coefficient is 0.304, indicating that similar nodes (according to degree) tend to connect to each other in this network.

3 Review of Centrality Measures

One of the central challenges in the structural analysis of a network is determining its centrality [14]. However, the identification of central elements is inherently ill-defined, as there is no unique definition of importance [15]. The notion of importance depends largely on the problem statement and the nature of the network under investigation. Consequently, a large number of centrality measures have been proposed in the literature. Over the past six decades, researchers have developed various methods to address this challenge and identify the most influential nodes in a network [16]. The earliest formal definition of a centrality measure dates back to the late 1940s [17]. Since then, a diverse set of centrality notions has emerged. In one context, the node with the highest degree may be considered most central; in another, the node that is closest on average to all others; and in yet another, the node that most effectively facilitates communication between other nodes. Thus, numerous approaches exist, each offering a suitable notion of centrality for a given context.

To bring order to this diversity, Bloch and colleagues [18] introduced criteria for classifying centrality-determination methods, suggesting that three distinct nodal statistics differentiate among centrality approaches. This raises a fundamental question: does a centrality method operate on paths, on walks, or on the shortest-path concept? In essence, centrality assessment involves evaluating the relative importance of nodes for a specific purpose, and determining which of the three nodal statistics is more relevant to that purpose.

A more comprehensive classification is provided by Das et al. [1], which categorizes centrality measures into six types based on their underlying principles: reachability, shortest path, feedback, current flow, random process, and vitality. This classification offers a structured framework for selecting appropriate centrality measures according to network characteristics and research objectives.

This section introduces several of the most common and widely used centrality measures in network analysis. We will apply these established methods in a subsequent section to assess the centrality of nodes within the previously unstudied WIPAN network. Our selection includes fundamental metrics-degree, closeness, and betweenness centrality-alongside more sophisticated algorithms such as PageRank, Katz, and eigenvector centrality. To ensure a holistic evaluation, we also incorporate advanced hybrid measures like Laplacian and second-order centrality. Applying this comprehensive, multi-metric framework to the novel WIPAN structure allows for a robust and nuanced analysis from multiple theoretical perspectives.

3.1 Degree Centrality

The simplest and perhaps oldest approach to identifying the central node of a network is to select the node with the highest degree, i.e., the node with the most neighbors, which is referred to as degree centrality [16]. Although this method is very straightforward, it remains logical and acceptable in many applications. A node with the highest degree can have the most connections and participate in a larger number of network interactions, making it appear more central. One of the most important advantages of this measure is its simplicity and low computational cost [10]. Degree centrality can be normalized to lie between zero and one [18]. In this case, the degree centrality of a node v indicates the fraction of the total nodes that are neighbors of v. Accordingly, the degree centrality for node v is given by Equation 12.

$$\mathrm{Cent}_d(v) = \frac{d_v}{n-1} \tag{12}$$

The normalized weighted degree centrality of node v is obtained from Equation 13. The numerator of this equation is the weighted degree of node v, and the denominator is the sum of all edge weights in the network (E denotes the edge set).

$$\mathrm{Cent}_{dw}(v) = \frac{\mathrm{dw}_v}{\sum\limits_{(i,j)\in E} w_{(i,j)}} \tag{13}$$

3.2 Closeness Centrality

Degree centrality provides a fast and simple method for determining centrality in many applications. However, it does not discriminate precisely among nodes, since the centrality of all nodes with the same degree is equal (in unweighted networks). Moreover, the node with the highest degree is not necessarily the central node of the network. For example, in a network, a peripheral node may have a high degree, while being distant from other nodes and thus having little influence on communications. Naturally, one would expect the network center to be as close as possible to all nodes; in other words, the center should be, if feasible, near the rest of the network [17]. This leads to the definition of closeness centrality. This measure defines centrality based on the notion of distance. Closeness centrality in a network is computed with respect to the shortest path between two nodes. To compute the closeness centrality of node v, one uses the reciprocal of the average distance from v to the other $n-1$ nodes in the network. Equation 14 shows the calculation of closeness centrality for node v [18]. In this equation, $dist_{(u,v)}$ denotes the length of the shortest path between nodes u and v, and $n-1$ is the number of nodes reachable from v.

$$\mathrm{Cent}_c(v) = \frac{n-1}{\sum\limits_{u=1}^{n-1} \mathrm{dist}_{(u,v)}} \tag{14}$$

Closeness centrality measures how long it takes for information to disseminate from a given node v to the rest of the network. A larger value of closeness centrality indicates greater centrality of the node. Compared with degree centrality, closeness centrality does not focus on the degree of a node; instead, it uses the distances from a node to all other nodes as the centrality criterion. In its weighted version, closeness centrality incorporates edge weights by using the weighted shortest-path distances in its calculations.

3.3 Betweenness Centrality

Although degree and distance can often indicate centrality, in some cases the centrality of a node is determined by the role it plays in connecting other nodes. For example, a node that functions as a bridge between two parts of a network may be considered central. In such a case, the shortest paths between many pairs of nodes pass through this node, and the absence of this node could even disconnect the network. The betweenness centrality concept views a node's centrality in terms of its position in the shortest paths between other nodes. In other words, how much control (i.e., influence) a node v has over the flow of information among other nodes determines its centrality. Betweenness centrality was developed independently by Anthonisse and Freeman [19, 20]. The betweenness centrality of node v is given by the sum over all pairs of nodes of the fraction of shortest paths between those pairs that pass through v, divided by the total number of shortest paths in the network [21]. Equation 15 is used to compute betweenness centrality.

$$\mathrm{Cent}_b(v) = \sum_{s \neq t \neq v} \frac{\sigma_{(s,t)}(v)}{\sigma_{(s,t)}} \tag{15}$$

In the above equation, V denotes the set of nodes, $\sigma_{(s,t)}$ is the number of shortest paths between two given nodes s and t, and $\sigma_{(s,t)}$ is the number of those shortest paths between s and t that pass through v (with v distinct from s and t). If $s = t$, then $\sigma_{(s,t)} = 1$, and if $v \in s, t$, then $\sigma_{(s,t)}(v) = 0$ [22].

Betweenness centrality treats all shortest paths (geodesics) with equal weight, regardless of the distance between the two nodes or the number of geodesics connecting them. A weighted variant of betweenness, known as godfather centrality, has been introduced [23]. In this approach, the weight is derived from the number of neighbors of v that are not directly connected to each other but become connected through v. Additionally, there are methods that incorporate the geodesic distance between nodes as weights when computing their betweenness [24].

3.4 Eigenvector Centrality

In a network, a high-degree node may have neighbors of low importance, which can diminish its overall centrality. A node with many connections might be deemed less central if it links to comparatively unimportant nodes. To address this limitation, eigenvector centrality generalizes degree centrality by considering both direct and indirect connections between nodes [15]. This measure assigns centrality to a node based on the centrality of its neighbors [25]. The eigenvector centrality of node v was redefined and applied for network-link analysis by Bonacich [26]. Random walks are used to compute this measure. This approach is equivalent to studying the stationary distribution of a Markov chain defined by a random walk on the network [16]. The eigenvector centrality of node v_i is given by the limit as $k \to \infty$ of the ratio between the number of length-k random walks that originate from v_i and the total number of length-k random walks in the network [10]. The mentioned ratio can be derived from the principal eigenvector of the network's adjacency matrix, such that the eigenvector centrality of node v_i corresponds to the ith entry in the principal eigenvector of the adjacency matrix A. The eigenvector centrality of node v_i is obtained from Equation 16, where A is the adjacency matrix of the network, φ_1 is the principal eigenvector of A, and λ_1 is the principal eigenvalue of A [10]. In this measure, the weight is interpreted as the connection strength.

$$\mathrm{Cent}_{ev}(v_i) = (\varphi_1)_i = \left(\frac{1}{\lambda_1} A \varphi_1\right)_i \tag{16}$$

Recently, the concept of Laplacian eigenvector centrality has been introduced by Shimono [27]. In this method, as with eigenvector centrality, spectral properties of the matrix are used. However, instead of employing the principal eigenvector of the adjacency matrix, the principal eigenvector of the Laplacian matrix is utilized. The authors claim that this approach offers high scalability and robustness.

3.5 Katz Centrality

Katz centrality is a generalization of eigenvector centrality introduced by Katz [28] in 1953. Like eigenvector centrality, Katz centrality assigns a node's importance based on the centrality of its neighbors. Katz centrality for a node is based on counting all paths that lead to the target node, with shorter paths receiving greater weight and longer paths receiving smaller weight. The Katz centrality of node v is computed from Equation 17.

$$\text{Cent}_{katz}(v) = \alpha \sum_{u \in N(v)} A_{(u,v)} \text{Cent}_{katz}(u) + \beta \quad \left(\alpha < \frac{1}{\lambda}\right) \tag{17}$$

In the above equation, A denotes the adjacency matrix of the network with its principal eigenvalue λ. The parameter β controls the initial value of the centrality, often set to 1 or a constant vector. It ensures even isolated nodes get non-zero centrality. α is a damping factor, typically chosen to be smaller than the inverse of the largest eigenvalue of A [29]. Katz centrality measures the relative importance of a node by counting its immediate neighbors (degree) and all other nodes that are connected to these immediate neighbors, i.e., it considers centrality beyond the immediate neighborhood and beyond the node's degree. The above equation can be solved and reformulated as Equation 18, in which k is the walk length and $(A^k)_{v,j}$ is the number of walks of length k from v to j. A smaller α behaves like degree centrality, while a larger α behaves like eigenvector centrality.

$$\text{Cent}_{katz}(v) = \beta \sum_{k=0}^{\infty} \alpha^k \sum_{j=1}^{n} (A^k)_{(v,j)} \tag{18}$$

Katz centrality is one of the commonly used centrality measures, particularly effective in sparse networks where the number of edges is small. Recently, an interesting approach [30] has been proposed to determine Katz centrality in very dense networks, achieving high efficiency like sparse networks while ranking nodes based on Katz centrality. In this approach, the complement of the dense network is used to compute Katz centrality. Thus, this centrality measure can be computed with very high efficiency in both sparse and dense networks. In the weighted version of Katz centrality, the weight is interpreted as the connection strength.

3.6 Load Centrality

The load centrality was introduced for the first time in [31]. It uses the concept of the shortest path to determine central nodes. It measures how much traffic (load) a node carries when every pair of nodes exchanges one unit of flow along the shortest paths, assuming that flow can be split across multiple shortest paths. Suppose that in a network, traffic is sent between all pairs of nodes along their shortest paths and divided evenly. The load centrality of node v is the total traffic that passes through this node if everyone communicates via shortest paths and divides traffic evenly if more than one shortest path exists between two nodes. The centrality of a node is determined by the total amount of traffic(load) that traverses it. To compute the load centrality for node v, one uses the ratio of the amount of traffic that passes through v to the total amount of traffic sent from s to t. It is computed from the following Equation:

$$\text{Cent}_{lo}(v) = \sum_{s \neq t \neq v} \frac{f_{(s,t)}(v)}{f_{(s,t)}} \tag{19}$$

where $f_{(s,t)}$ is the total amount of flow sent from s to t (typically one unit), and $f_{(s,t)}(v)$ is the amount of that flow that passes through v, split evenly among all shortest paths from s to t. In this respect, load centrality resembles betweenness centrality [32] and is numerically close to it. In fact, betweenness centrality counts paths, but load centrality counts traffic moving along those paths. In the weighted version, load centrality is defined exactly as in the unweighted case, except that the shortest paths are computed using edge weights.

3.7 Harmonic Centrality

Harmonic centrality was introduced by Boldi [16]. Inspired by the work of Marchiori and Latora [33], they employed the harmonic mean of the shortest distances to compute centrality. The harmonic mean can address the issue of isolated nodes in a network, since the distance between two disconnected nodes is considered infinite, and the inverse of infinity (i.e., zero) is used in the harmonic mean calculation. The harmonic centrality of a node v is the sum of the inverse of the shortest distances from v to all other nodes in the network and is obtained from Equation 20 [18]

$$\text{Cent}_h(v) = \frac{1}{n-1} \sum_{u \neq v} \frac{1}{\text{dist}_{(u,v)}} \tag{20}$$

In this equation, $dist_{(u,v)}$ denotes the length of the shortest path between nodes u and v. In simple networks, harmonic centrality is entirely correlated with closeness centrality, with the distinction that it also accounts for pairs of nodes between which no path exists, and for this reason it is preferred to closeness centrality [16].

3.8 Laplacian Centrality

Laplacian centrality was introduced in [34]. Its name derives from the use of the network Laplacian matrix in computing this centrality measure. First, the concept of the network Laplacian energy-which can be viewed as a form of network entropy-is obtained via Equation 21. It is defined as the sum of the squares of the eigenvalues of the network Laplacian matrix.

Then the Laplacian centrality of node v_i is determined by the decrease in the network's Laplacian energy resulting from removing node v_i from the network [34]. In the following equations, λ_i denotes the i-th eigenvalue of the network Laplacian matrix, and $EL(G)$ is the Laplacian energy of the network, while $EL(G_i)$ is the Laplacian energy after removing node v_i. As is evident from Equation 22, a node's centrality is proportional to the magnitude of ΔE; that is, the larger the drop in Laplacian energy caused by removing a node, the more central that node is. The term $EL(G)$ in the denominator of this ratio is used to normalize the centrality measure.

$$E_L(G) = \sum_{i=0}^{n} \lambda_i^2 \tag{21}$$

$$\text{Cent}_{la}(v_i) = (\Delta E)_i = \frac{E_L(G) - E_L(G_i)}{E_L(G)} \tag{22}$$

The calculation of Laplacian centrality is centered on the term $(\Delta E)_i$, which can be derived from node degrees using Equation 23 [35]. This measure provides insight into the structural connections and local density around node v, extending beyond its immediate neighbors. Specifically, Laplacian centrality incorporates information from nodes within a two-step neighborhood, including the node itself, its direct neighbors, and neighbors at distance two. Unlike degree centrality, which relies on local information, or closeness and betweenness centralities, which depend on global network properties, Laplacian centrality functions as an intermediary metric by integrating both local and global structural characteristics [34]. Consequently, and as suggested by Equation 23, a strong correlation is expected between Laplacian centrality and degree centrality.

$$(\Delta E)_i = d_i^2 + d_i + 2 \sum_{j \in N(i)} d_j \tag{23}$$

More advanced methods are designed based on Laplacian centrality. Zhu and colleagues [36] introduced an enhanced version of Laplacian centrality, whose computation is the same as that of standard Laplacian centrality except that, wherever degree was used in the original calculation, the improved method uses Laplacian centrality.

$$\text{Cent}_{ila}(v_i) = \text{Cent}_{la}(v_i)^2 + \text{Cent}_{la}(v_i) + 2 \sum_{j \in N(i)} \text{Cent}_{la}(v_j) \tag{24}$$

Evaluations on nine real networks show that this approach outperforms the original version and six classical centrality measures in terms of ranking accuracy and identifying top nodes. In addition, its computational complexity does not increase substantially compared to the original version.

3.9 Current-Flow Closeness Centrality (CFCC)

This centrality measure is based on the "information" contained in all possible paths between pairs of nodes. It is a variant of closeness centrality based on effective resistance between nodes in a network [37]. In closeness centrality calculation, geodesic distance (shortest path distance) is used as the measure of distance. In CFCC, resistance distance is used as a measure of distance between nodes. In fact, CFCC is a modified form of closeness centrality that replaces the use of shortest-path distances with resistance distances. The resistance distance $\Omega_{u,v}$ between a pair of nodes in a connected component of a network can be calculated as follows [38]:

$$\Omega_{(u,v)} = \frac{\det L(G-u-v)}{\det L(G-u)} \tag{25}$$

Let L be the Laplacian matrix of the network, let $L(G-u)$ be the matrix resulting from the removal of the uth row and column of L, and let $L(G-u-v)$ be the matrix resulting from the removal of both the uth and vth rows and columns of L. The following equation is used to calculate the CFCC of node v:

$$\text{Cent}_{cfc}(v) = \frac{n-1}{\sum_{u=1}^{n-1} \Omega_{(u,v)}} \tag{26}$$

By considering the distance between two nodes as electrical resistance, CFCC assesses how readily current can flow from a node to others. For a node v, the CFCC is obtained by aggregating the amount of current that can be delivered from v to all other nodes under the electrical current-flow model with a one-ohm resistor on all edges. In weighted networks, the network's edge weights are interpreted as conductance (the inverse of resistance). Higher weight means higher conductance and lower resistance. Then weights represent the capacity or strength of the edge, indicating how much flow can pass or how strongly two nodes are connected.

3.10 Current-Flow Betweenness Centrality (CFBC)

Instead of counting the shortest paths (as in standard betweenness), current-flow betweenness centrality considers all possible paths and how easily flow can pass through a node. It uses an electrical current model for information spreading [37]. It is also known as random-walk betweenness centrality [39]. In fact, it computes how much, on average, a node lies on the paths between all pairs of nodes when flow is allowed to take multiple routes proportionally to conductance. For each pair of distinct nodes (s,t), we consider how the injected current from s to t splits through the network; the contribution of v is the proportion of that current that passes through v, and we sum these contributions over all pairs. The CFBC for a node v is typically defined as

$$\text{Cent}_{cfb}(v) = \sum_{s \neq t \neq v} \frac{I_{(s,t)}(v)}{I_{(s,t)}} \tag{27}$$

where $I_{(s,t)}$ is the total current entering the network at source s and leaving at sink t, and $I_{(s,t)}(v)$ is the amount of that current that passes through node v. In weighted networks, edge weights represent the capacity or strength of the edge, indicating how much flow can pass or how strongly two nodes are connected.

3.11 Second-order Centrality

The second-order centrality of a given node is the standard deviation of the return times to that node of a perpetual random walk on the network [40]. A perpetual random walk is the random walk that continues indefinitely, moving from node to node according to transition probabilities (often based on edge weights or uniform choices among neighbors).

The return time for node v is the number of steps required for a random walker, starting from v, to return to v. The variability of these return times is measured by their standard deviation across runs. The lower values of second-order centrality indicate higher centrality. The second-order centrality for node v is obtained from the following equation, where r_v^k denotes the kth return time to node v.

$$\text{Cent}_{so}(v) = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n} r_v^k - \left(\frac{1}{n-1}\sum_{k=1}^{n} r_v^k\right)^2} \tag{28}$$

To compute CFBC, Newman [39] used a simple random walk on the network, where at each step the walker moves to a uniformly random neighbor. This yields a stationary distribution proportional to node degree, $\pi_i = d_i/(2m)$, so higher-degree nodes are visited more often. In contrast, Kermarrec [40] proposed an unbiased random-walk strategy in which, after a sufficient number of steps, the probability of being at any network node becomes uniform, $\pi_i = 1/n$. Unbiasing the walk ensures that variations in return times reflect only the nodes' topological importance, rather than local factors such as degree. Accordingly, the random walk from node i to a neighbor j uses a transition probability proportional to d_i/d_j rather than the degree-proportional probability $1/d_i$; the walker progresses to the chosen neighbor with a probability that depends on both node degrees.

3.12 PageRank Centrality

PageRank is an algorithm that assigns a centrality score to each node in a network, reflecting its importance based on the structure of incoming links. It evaluates the probability of visiting each node by a random walker and uses it as the centrality [15]. The algorithm was originally developed to rank web pages, where links represent citations or references [41]. Conceptually, a random surfer traverses the network by following outgoing edges, and the PageRank of a node corresponds to the long-run fraction of time the surfer spends at that node. The damping factor α introduces random jumps, ensuring convergence and mitigating the impact of poorly connected areas. The core intuition is that being linked to important pages (or nodes) increases one's own importance. Although PageRank is defined for directed networks, it can be applied to undirected networks by treating each undirected edge as bidirectional and applying the standard PageRank. In undirected networks, PageRank relates to eigenvector centrality but remains a distinct measure because it is grounded in a stochastic process with teleportation rather than a purely eigenvector criterion. The PageRank vector is computed iteratively. In each iteration, the vector is updated according to the following equation

$$\boldsymbol{\pi}^{(t+1)} = \alpha \mathbf{M}^T \boldsymbol{\pi}^{(t)} + (1-\alpha)\mathbf{v} \tag{29}$$

where α is the damping factor, M is the (column-stochastic) transition matrix derived from the network, and v is the teleportation vector (often uniform, $v_i = 1/n$). The procedure is repeated until convergence, yielding the stationary PageRank vector π that reflects the long-run visit distribution across nodes [42].

3.13 Concluding Remarks

The centrality measures introduced above can be understood within a broader organizational framework. Das et al. [1] provide a comprehensive categorization of centrality measures as mathematical tools for identifying important elements in network analysis. According to this study, centrality measures fall into six distinct categories based on their underlying principles:

- Reachability-based measures (e.g., degree centrality, closeness centrality) indicate whether nodes are directly or indirectly connected and assess the ability of vertices to reach all others.
- Shortest path-based measures (e.g., betweenness centrality) identify paths between vertices and determine the shortest route from source to sink.
- Feedback-based measures (e.g., eigenvector centrality) operate on the principle that a node is important if its neighbors are important.
- Current flow-based measures (e.g., current flow

betweenness centrality, current flow closeness centrality) assume that information flow follows the behavior of electrical current in a network.
- Random process-based measures (e.g., PageRank centrality, second-order centrality) are calculated through random walks, where a vertex randomly chooses edges to reach another vertex.
- Vitality-based measures (e.g., Laplacian centrality) evaluate the difference in a network function with and without a given vertex.

This categorization offers a structured framework for selecting appropriate centrality measures based on the nature of the network and the specific research problem. However, selecting the most appropriate measure for a given application remains a challenge. As discussed in [15], since the notion of importance depends largely on the problem statement and the network's nature, the choice of centrality measure is inherently application-dependent. To address this, Meshcheryakova [15] presents 11 axioms, including Density, Size, Anonymity, Bridge, Monotonicity, Fairness, Endpoint Increase, Balanced Contributions, Add Edge Distance, Cut-Vertex Additivity, and Top Node, to guide the selection process. The study provides an overview of existing centrality axioms and reviews current results on verifying these axioms for classical centrality measures.

4 Analysis of Centrality Measures in WIPAN

As established in the preceding section, network centrality can be conceptualized through multiple paradigms, each offering distinct interpretations of a node's importance. The selection of centrality measures should therefore align with the specific topological or functional attributes relevant to the analytical context. In this study, twelve established centrality measures were applied to the WIPAN network to generate a comprehensive, multi-faceted ranking of provincial nodes.

4.1 Distribution of Centrality Values

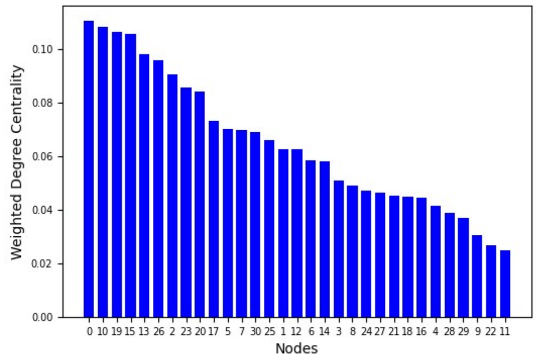
The results of applying these twelve centrality measures are presented as histograms in Figure 4a through Figure 4ℓ, providing an initial overview of how centrality is dispersed across the network under each metric.

- Degree Centrality (Figure 4a): Nodes 0, 10, and 19 emerge as the most central, with their weighted centrality influenced by both the number and geographic proximity of their connections. While node 10 has the highest degree (9), node 0 achieves superior weighted centrality due to closer average distances to its seven neighbors. Conversely, nodes 11, 22, and 9 exhibit the lowest centrality, attributable to their low degree and peripheral geographic positions.
- Closeness & Harmonic Centrality (Figure 4b & Figure 4g): These distance-based metrics identify nodes 25, 23, 30 (closeness) and 26, 25, 23 (harmonic) as most central, highlighting provinces optimally positioned for efficient access to the rest of the network.
- Betweenness & Load Centrality (Figure 4c & Figure 4f): Both metrics pinpoint nodes 23, 30, and 10 as critical bridges controlling shortest-path flows. Their high centrality underscores their role as indispensable conduits for network connectivity.
- Eigenvector & Katz Centrality (Figure 4d & Figure 4e): These measures, which account for a node's connections to other central nodes, highlight nodes 0, 13, 26 (eigenvector) and 0, 15, 10 (Katz). This indicates that centrality is not only an attribute of individual nodes but also propagates through influential regional clusters.
- Current-Flow Centralities (Figure 4i & Figure 4j): Modeling flow via all possible paths, these measures consistently identify nodes 10, 0, and 15 as most central, demonstrating robustness when considering alternative routing.
- Laplacian Centrality (Figure 4h): This measure, sensitive to a node's impact on network energy, also highlights nodes 0, 15, and 10, aligning with current-flow results and emphasizing their structural criticality.
- Second-Order Centrality (Figure 4k): Based on the regularity of return times for a random walker, this method identifies nodes 10, 0, and 15 as most central, suggesting their positions naturally attract and retain flow.
- PageRank (Figure 4ℓ): Simulating a random surfer, this algorithm ranks nodes 10, 19, and 15 highest, indicating their likely prominence in processes of random exploration and preferential visitation.

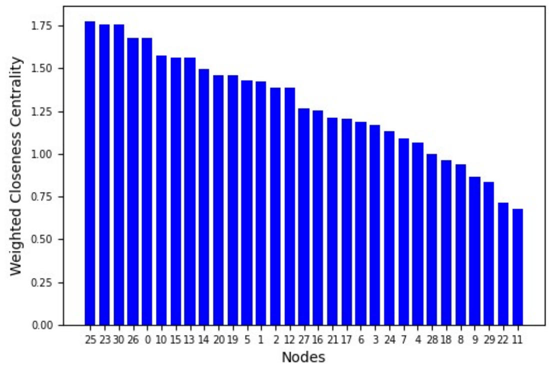
A consistent pattern across most measures is the low centrality of eastern and southern provinces (e.g., nodes 11, 22, 9), attributable to their larger inter-capital distances. In contrast, provinces in the west, north, and central regions, benefiting from higher adjacency and shorter distances, consistently receive higher centrality scores.

4.2 Comparative Analysis and Correlation

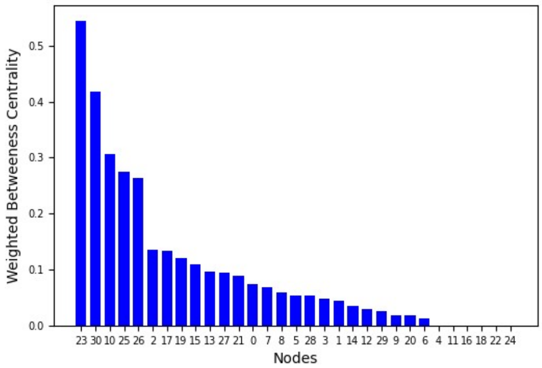
The numerical centrality values for all nodes and measures are summarized in Table 2. The correlation matrix (Figure 5) elucidates the relationships between



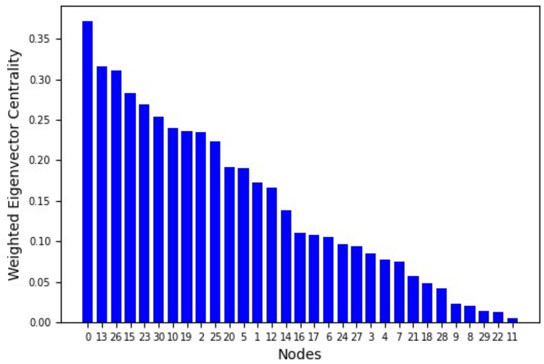
(a) Degree Centrality.



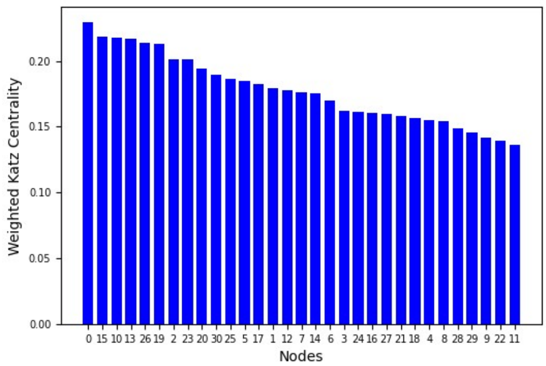
(b) Closeness Centrality.



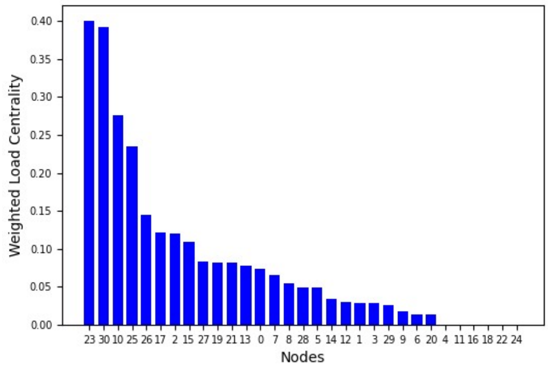
(c) Betweenness Centrality.



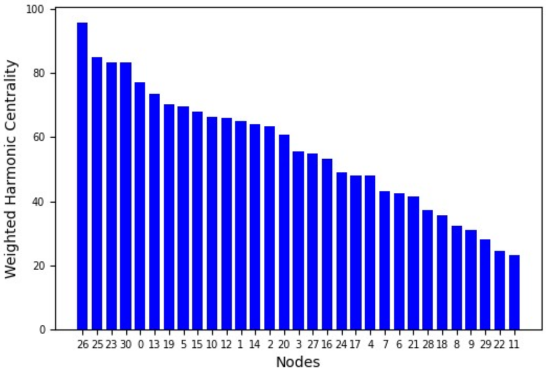
(d) Eigenvector Centrality.



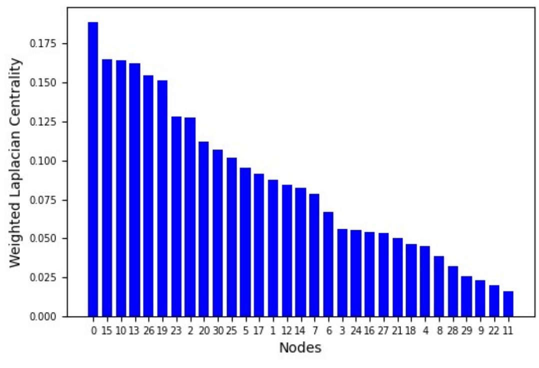
(e) Katz Centrality.



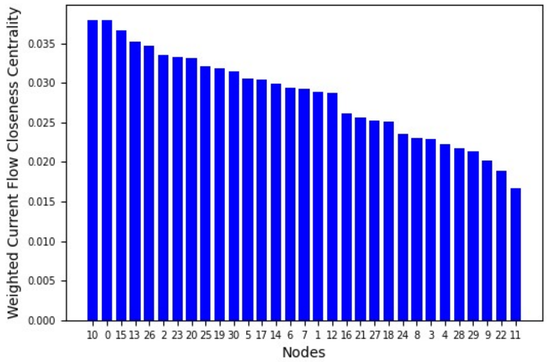
(f) Load Centrality.



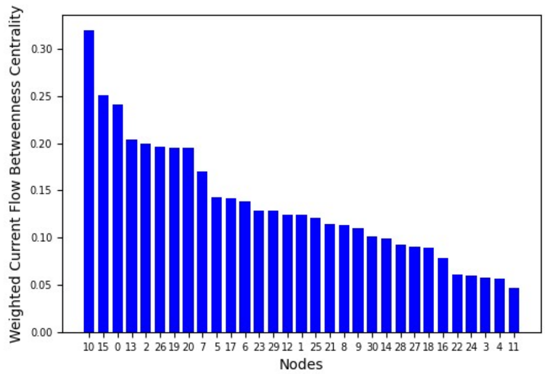
(g) Harmonic Centrality.



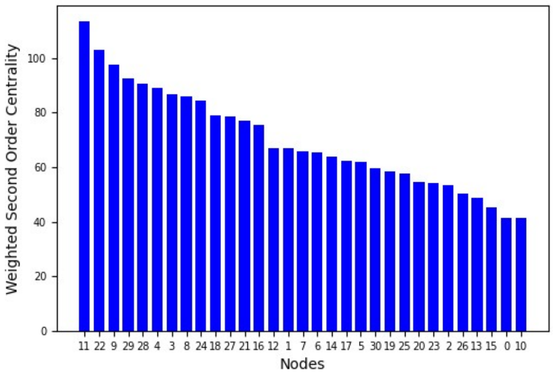
(h) Laplacian Centrality.



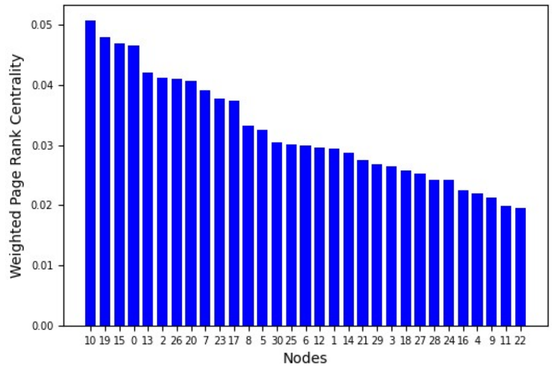
(i) Current-Flow Closeness Centrality.



(j) Current-Flow Betweenness Centrality.



(k) Second-order Centrality.



(ℓ) PageRank Centrality.

Figure 4. Histogram Plots of Centrality Measures for the WIPAN.

these diverse metrics, revealing a pronounced clustering into three distinct conceptual groups:

- A tightly coupled degree and recursive influence cluster (Degree, Katz, Laplacian, Eigenvector, Current-Flow Closeness; $r > 0.94$), indicating that local connectivity strongly predicts global influence and flow-based importance in WIPAN.
- A distance-based cluster comprising Closeness and Harmonic centrality ($r = 0.97$).
- A bridging cluster containing Betweenness and Load centrality ($r = 0.98$), which is notably isolated from the other two clusters.

A critical finding is the extremely low correlation ($r = 0.33$) between standard Betweenness and Current-Flow Betweenness centrality. This stark divergence underscores that identifying critical nodes for shortest-path control versus robustness in all possible pathways are fundamentally different tasks in a geographically weighted network like WIPAN. Consequently, the choice between these measures must be explicitly justified by the application context-security of critical routes versus overall network reliability.
PageRank shows a stronger affiliation with the flow-based measures (CFBC, Katz) than with pure eigenvector-based prestige, while Second-Order centrality demonstrates moderate correlations with all clusters, positioning it as a hybrid metric capturing a balanced blend of connectivity, proximity, and bridging roles.

4.3 Consensus Ranking and Methodological Evaluation

To synthesize the findings from all twelve methods, a consensus ranking [43, 44] was derived by averaging the ordinal rank of each node across all measures. This aggregated ranking, along with the individual ranks, is presented in Table 3.
Most Central Nodes: The Consensus ranking identifies Node 10 as the most central node in WIPAN. It emerges as the unequivocal leader, securing first place in four measures (CFBC, CFCC, Second-Order, PageRank) and ranking no lower than tenth. It is followed by nodes 0, 15, 26, and 13. The high consensus rank of Node 23 (Tehran)-particularly bolstered by its top rank in Betweenness and second rank in Closeness-objectively justifies its historical role as a national capital, functioning as a critical hub for shortest-path connectivity and proximate access. However, its much

	Deg.	Clo.	Bet.	Kat.	Loa.	Har.	Eig.	Lap.	CFB	CFC	SO	PR
Deg.	1	0.796	0.489	0.989	0.474	0.765	0.906	0.984	0.883	0.948	0.608	0.963
Clo.	0.796	1	0.674	0.859	0.664	0.967	0.915	0.85	0.564	0.888	0.76	0.653
Bet.	0.489	0.674	1	0.524	0.983	0.656	0.569	0.517	0.329	0.532	0.65	0.431
Kat.	0.989	0.859	0.524	1	0.508	0.833	0.956	0.998	0.843	0.965	0.655	0.919
Loa.	0.474	0.664	0.983	0.508	1	0.62	0.548	0.501	0.339	0.533	0.714	0.424
Har.	0.765	0.967	0.656	0.833	0.62	1	0.916	0.827	0.491	0.824	0.705	0.6
Eig.	0.906	0.915	0.569	0.956	0.548	0.916	1	0.96	0.706	0.916	0.698	0.774
Lap.	0.984	0.85	0.517	0.998	0.501	0.827	0.96	1	0.843	0.955	0.642	0.91
CFB	0.883	0.564	0.329	0.843	0.339	0.491	0.706	0.843	1	0.838	0.438	0.931
CFC	0.948	0.888	0.532	0.965	0.533	0.824	0.916	0.955	0.838	1	0.696	0.882
SO	0.608	0.76	0.65	0.655	0.714	0.705	0.698	0.642	0.438	0.696	1	0.514
PR	0.963	0.653	0.431	0.919	0.424	0.6	0.774	0.91	0.931	0.882	0.514	1

Figure 5. Pairwise Correlations Among the 12 Centrality Measures Computed for the WIPAN.

lower rank in CFBC (#13) highlights the conceptual divergence between controlling shortest paths and facilitating system-wide flow.

Least Central Nodes: Node 11 is consistently identified as the least central, ranking last in 11 of the 12 measures. Nodes 4, 29, 9, and 22 also occupy the peripheral positions in the consensus ranking, consistent with their geographic and topological isolation.

Deviation Analysis: The final row of Table 3 quantifies the standard deviation of each method's ranking from the consensus. This analysis provides critical insight for method selection:

- Katz and Laplacian centralities exhibit the smallest deviation (82), demonstrating that they effectively balance local and global network properties to produce a robust and representative centrality assessment. The computational efficiency of Laplacian centrality, derived from degree information, makes it particularly practical.
- Degree Centrality, despite its simplicity, shows relatively low deviation and high correlation with many advanced methods, confirming its enduring utility as a fast, reliable heuristic.
- Betweenness and Load centralities show the largest deviation (1048, 1126), as they specialize in identifying bridge nodes-a specific centrality role not emphasized by other metrics. Their strong mutual correlation but weaker correlation with other measures confirms their unique conceptual focus.
- Second-Order Centrality shows a low deviation (114), confirming its effectiveness as a balanced hybrid metric despite its unique foundation in random walk return-time variance.

4.4 Practical Implications

Based on the consensus ranking derived from 12 different centrality measures on WIPAN, we can now provide specific, practical recommendations.

4.4.1 Identifying Transit Bottlenecks (Chokepoints)

Province 10 (consensus rank 1) consistently ranks at the top in betweenness centrality and current-flow-based measures. This means that many shortest-distance paths between other province pairs pass through Province 10. Any disruption there (e.g., natural disaster, road closure, or security incident) would force traffic onto significantly longer alternative routes. This finding suggests the following:

- Locate transit monitoring stations, customs clearance hubs, and strategic military checkpoints in Province 10.
- Prioritize this province for road and railway reinforcement to prevent national paralysis.

4.4.2 Emergency and Disaster Response Team Placement

Province 0 (rank 2) and Province 15 (rank 3) show very high closeness centrality and eigenvector centrality, meaning they have the shortest average distance to all other provinces and are connected to other well-connected provinces. Accordingly, practical recommendations include:

- Position national search-and-rescue headquarters and emergency medical teams in Province 0.
- Place secondary response bases in Province 15 and Province 26 (rank 4) to cover the entire country with minimal response time.

Table 2. Node Centrality Scores for the 31 Iranian Provinces based on 12 Network Measures.

Province No	Deg. Cent.	Clo. Cent.	Bet. Cent.	Kat. Cent.	Loa. Cent.	Har. Cent.	Eig. Cent.	Lap. Cent.	CFB Cent.	CFC Cent.	SO Cent.	PR Cent.
0	0.111	1.675	0.075	0.230	0.074	77.264	0.372	0.189	0.241	0.038	41.380	0.047
1	0.063	1.421	0.045	0.179	0.029	65.091	0.173	0.088	0.124	0.029	67.086	0.029
2	0.090	1.387	0.135	0.202	0.120	63.560	0.234	0.127	0.200	0.034	53.505	0.041
3	0.051	1.167	0.048	0.162	0.029	55.520	0.085	0.056	0.057	0.023	86.630	0.026
4	0.041	1.068	0.000	0.155	0.000	48.052	0.077	0.045	0.056	0.022	89.189	0.022
5	0.070	1.430	0.055	0.185	0.049	69.676	0.190	0.095	0.143	0.031	62.017	0.032
6	0.058	1.185	0.014	0.170	0.014	42.598	0.105	0.067	0.139	0.029	65.540	0.030
7	0.070	1.090	0.070	0.176	0.066	43.106	0.074	0.079	0.170	0.029	65.761	0.039
8	0.049	0.938	0.059	0.155	0.055	32.280	0.020	0.038	0.114	0.023	85.915	0.033
9	0.031	0.867	0.018	0.142	0.018	31.075	0.023	0.023	0.110	0.020	97.487	0.021
10	0.108	1.572	0.307	0.218	0.275	66.533	0.240	0.164	0.320	0.038	41.275	0.051
11	0.025	0.677	0.000	0.137	0.000	23.240	0.005	0.016	0.047	0.017	113.640	0.020
12	0.063	1.384	0.030	0.178	0.030	66.039	0.167	0.084	0.124	0.029	67.211	0.030
13	0.098	1.561	0.097	0.217	0.078	73.427	0.316	0.163	0.203	0.035	48.897	0.042
14	0.058	1.495	0.034	0.175	0.034	64.108	0.138	0.082	0.099	0.030	63.785	0.029
15	0.106	1.563	0.110	0.219	0.109	68.029	0.283	0.165	0.251	0.037	45.168	0.047
16	0.044	1.256	0.000	0.160	0.000	53.156	0.111	0.054	0.079	0.026	75.428	0.023
17	0.073	1.206	0.133	0.182	0.122	48.169	0.108	0.092	0.141	0.030	62.265	0.037
18	0.045	0.960	0.000	0.156	0.000	35.513	0.048	0.046	0.090	0.025	78.909	0.026
19	0.106	1.459	0.121	0.213	0.082	70.165	0.236	0.152	0.196	0.032	58.444	0.048
20	0.084	1.461	0.018	0.194	0.014	60.691	0.191	0.112	0.195	0.033	54.693	0.041
21	0.045	1.211	0.090	0.158	0.082	41.403	0.057	0.051	0.114	0.026	77.144	0.028
22	0.027	0.716	0.000	0.139	0.000	24.428	0.013	0.020	0.061	0.019	103.046	0.020
23	0.085	1.755	0.545	0.202	0.401	83.326	0.269	0.128	0.129	0.033	54.389	0.038
24	0.047	1.132	0.000	0.161	0.000	48.896	0.096	0.055	0.060	0.024	84.394	0.024
25	0.066	1.776	0.275	0.186	0.234	85.120	0.224	0.102	0.121	0.032	57.553	0.030
26	0.096	1.680	0.264	0.214	0.145	95.907	0.311	0.155	0.196	0.035	50.267	0.041
27	0.046	1.263	0.094	0.160	0.084	55.040	0.093	0.054	0.090	0.025	78.465	0.025
28	0.039	0.999	0.055	0.149	0.049	37.298	0.042	0.032	0.093	0.022	90.779	0.024
29	0.037	0.833	0.025	0.146	0.025	27.991	0.015	0.026	0.128	0.021	92.609	0.027
30	0.069	1.755	0.417	0.190	0.391	83.326	0.254	0.107	0.101	0.031	59.512	0.030

4.4.3 New Capital City Selection

Province 0 (rank 2) and Province 10 (rank 1) are the strongest candidates. Province 0 has superior closeness centrality (rank 2 in that measure), minimizing average travel distance for citizens from all other provinces. Province 10 has unmatched betweenness centrality, making it highly accessible. Its implications are as follows:

- If the goal is equitable geographic access for all citizens, select Province 0 as the new administrative capital.
- If the goal is maximum control over national transit flows, select Province 10.

4.4.4 Economic Facility Location (Warehouses, Distribution Centers)

Province 10 (rank 1) and Province 13 (rank 5) exhibit high weighted degree centrality and Katz index, indicating many nearby provinces with short distances. This has several practical implications:

- Locate national food distribution centers, pharmaceutical warehouses, and e-commerce logistics

Table 3. Node rankings for the 31 Iranian provinces based on 12 centrality measures and the resulting consensus ranking.

Province No	Deg. Cent.	Clo. Cent.	Bet. Cent.	Kat. Cent.	Loa. Cent.	Har. Cent.	Eig. Cent.	Lap. Cent.	CFB Cent.	CFC Cent.	SO Cent.	PR Cent.	Average Rank	Consensus Rank
10	2	6	3	3	3	10	7	3	1	1	1	1	3.42	1
0	1	5	13	1	13	5	1	1	3	2	2	4	4.25	2
15	4	7	9	2	8	9	4	2	2	3	3	3	4.67	3
26	6	4	5	5	5	1	3	5	6	5	5	7	4.75	4
13	5	8	10	4	12	6	2	4	4	4	4	5	5.67	5
23	8	2	1	8	1	3	5	7	13	7	7	10	6.00	6
19	3	11	8	6	10	7	8	6	7	10	10	2	7.33	7
2	7	14	6	7	7	14	9	8	5	6	6	6	7.92	8
30	13	2	2	10	2	3	6	10	21	11	11	14	8.75	9
25	14	1	4	11	4	2	10	11	17	9	9	15	8.92	10
20	9	10	24	9	24	15	11	9	8	8	8	8	11.92	11
5	11	12	16	12	17	8	12	12	10	12	12	13	12.25	12
17	10	19	7	13	6	20	17	13	11	13	13	11	12.75	13
1	15	13	19	14	20	12	13	14	16	17	17	18	15.67	14
7	12	23	14	16	14	22	23	17	9	16	16	9	15.92	15
12	16	15	21	15	19	11	14	15	15	18	18	17	16.17	16
14	18	9	20	17	18	13	15	16	22	14	14	19	16.25	17
6	17	20	25	18	24	23	18	18	12	15	15	16	18.42	18
27	22	16	11	22	9	17	20	22	24	21	21	24	19.08	19
21	23	18	12	23	10	24	24	23	18	20	20	20	19.58	20
3	19	21	18	19	20	16	21	19	29	25	25	22	21.17	21
16	25	17	26	21	26	18	16	21	26	19	19	27	21.75	22
8	20	27	15	26	15	27	28	26	19	24	24	12	21.92	23
24	21	22	26	20	26	19	19	20	28	23	23	26	22.75	24
28	27	25	16	27	16	25	26	27	23	27	27	25	24.25	25
18	24	26	24	26	24	26	25	24	25	22	22	23	24.42	26
4	26	24	26	25	26	21	22	25	30	26	26	28	25.42	27
29	28	29	22	28	22	29	29	28	14	28	28	21	25.50	28
9	29	28	23	29	23	28	27	29	20	29	29	29	26.92	29
22	30	30	26	30	26	30	30	30	27	30	30	31	29.17	30
11	31	31	26	31	26	31	31	31	31	31	31	30	30.08	31
Deviation	144	497	1048	82	1126	621	284	82	818	114	114	420		

hubs in Province 10.

- Place regional perishable goods centers (e.g., fresh produce, cold storage) in Province 13 and Province 23 (rank 6).

4.4.5 Military and Strategic Facility Placement

Province 10 (rank 1) is ideal for a national defense command center due to its high betweenness (control over routes). Province 0 (rank 2) is better suited for a rapid-deployment force base due to its closeness to all regions. This finding suggests the following:

- Station central military headquarters and intelligence monitoring hubs in Province 10.
- Position quick-reaction ground forces in Province 0 to reach any border within hours.

4.4.6 Communication and Transport Hub Placement

Province 30 (rank 9) and Province 25 (rank 10) show balanced scores across multiple centrality measures, making them excellent candidates for secondary hub cities. Accordingly, practical recommendations include:

- Install national internet exchange points (IXP) and fiber-optic backbone nodes in Province 30.
- Develop intermodal transport terminals (rail, road, air) in Province 25.

4.4.7 Public Health Network Design

Province 23 (rank 6) has high closeness and low average distance to neighbors, ideal for a centralized blood bank or organ transplant coordination center. This finding suggests the following:

- Build a national vaccine storage facility in Province 23 to enable rapid distribution during pandemics.
- Locate a specialized trauma center in Province 19 (rank 7) for covering multiple provinces.

4.4.8 Periphery Identification for Targeted Investment

Province 11 (rank 31) ranks lowest in almost all centrality measures. This indicates long distances to other provinces and a minimal role in national shortest-path routes. This has several practical implications:

- This province is a candidate for special economic zone status with tax incentives to attract local development.
- Prioritize new highway construction connecting Province 11 to higher-centrality provinces to reduce isolation.

4.4.9 Environmental Monitoring

Province 2 (rank 8) shows high current-flow centrality, meaning it lies on many critical distance-minimizing paths. Its implication is as follows:

- Place air quality monitoring stations and hazardous material spill response teams in Province 2 to detect and contain pollution spreading across multiple provinces.

While high-centrality provinces are critical for network efficiency, low-centrality provinces such as 11, 22, 9, and 29 are equally vital for national resilience, equity, and untapped potential. These peripheral areas suffer from geographical isolation (e.g., Province 11) and long distances to central hubs, making them prime candidates for infrastructure investment, decentralization policies, and specific facilities that benefit from low centrality, such as prisons, hazardous material storage, military grounds, renewable energy farms, and data centers. From a resilience perspective, they serve as strategic reserves and redundancy hubs to avoid single points of failure. Additionally, they offer opportunities for environmental conservation, eco-tourism, targeted healthcare and education access (e.g., telemedicine and mobile clinics), infrastructure prioritization based on the centrality gap (e.g., building direct highways to core provinces), and disaster evacuation and relief staging areas when high-centrality provinces are disrupted. Table 4 and Table 5 summarize the practical implications of the results for both high-centrality and low-centrality provinces.

5 Conclusion and Future Works

This study contributes to the literature on geographical network analysis by proposing the WIPAN model and demonstrating its applicability through a consensus-based centrality ranking, offering both methodological insights and practical decision-support tools. The WIPAN is an undirected weighted network modeling Iran's provincial adjacency, with 31 nodes representing provincial capitals and 74 edges weighted by geographical distance. After characterizing the network's structure, twelve established centrality measures were applied to assess nodal importance. Although individual metrics highlighted different candidates-with nodes 0 and 10 each ranking highest in four measures-the consensus ranking derived from all twelve methods definitively identified Node 10 as the most central node in WIPAN. Conversely, Node 11 was consistently ranked as the least central, supported by last-place rankings in eleven of twelve measures. Peripheral nodes such as 22, 9, and 29 were also consistently ranked low, aligning with their geographically distant positions in the east and south of the country. While high-centrality provinces (e.g., 10, 0, 15) are optimal for efficiency-driven facilities such as national hubs, command centers, and transit bottlenecks, low-centrality provinces (e.g., 11, 22, 9, 29) are not merely 'disadvantaged.' Instead, they offer unique opportunities for decentralization, strategic redundancy, environmental conservation, and targeted development. A balanced national policy should therefore invest in connecting peripheral provinces to the core network while leveraging their isolation for specific applications that benefit from low centrality.

It should be acknowledged that the network analyzed in this study is relatively small in size, which limits the generalizability of the findings to larger or more complex networks. However, applying the same methodological framework to larger geographical datasets in future studies could help validate and extend these observations. Future research may also extend this work by applying additional centrality measures to further validate the consensus ranking, or by redefin-

Table 4. Summary of Applications of the Results.

Province	Consensus Rank	Application	Reason
10	1	Transit bottleneck / Choke-point	Highest betweenness & current-flow
0, 15	2, 3	Emergency response base	Highest closeness centrality
0	2	New capital (accessibility)	Best closeness
10	1	New capital (control)	Best betweenness
10, 13	1, 5	National warehouse / distribution	High weighted degree & Katz
10	1	Military command center	Control over routes
0	2	Rapid deployment force base	Shortest average distance to all
30, 25	9, 10	Secondary hub (telecom/transport)	Balanced centralities
23	6	Blood bank / vaccine storage	High closeness to neighbors
11	31 (lowest)	Targeted development (periphery)	Lowest centrality (needs investment)

Table 5. Recommendations for Low-Centrality Provinces.

Province	Consensus Rank	Primary Challenge	Key Application / Recommendation
11	31	Extreme isolation	Priority for new road/rail links; telemedicine hub; emergency reserve storage
22	30	Very low connectivity	Special economic zone; backup data center; mobile health clinics
9	29	Long distances to all	National park; eco-tourism; military training ground
29	28	Poor access to central services	Regional referral hospital; renewable energy farm; evacuation destination

ing edge weights using alternative data such as travel time, economic flows, or shared border length. Most importantly, the centrality profiles generated here can directly inform practical applications, including strategic infrastructure placement, emergency service planning, and regional development, transforming the WIPAN model into a useful practical tool for geographic and infrastructural decision-making.

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