An Optimal Traffic Distribution Method Supporting End-to-End Delay Bound

Touraj Shabanian, MassoudReza Hashemi, Behnaz Omoumi

Abstract— **Routing methods for optimal distribution of traffic in data networks that can also provide quality of service (QoS) to users is one of the challenges of recent research on next generation networks. The QoS requirement in most cases is an upper bound on end-to-end path delay. In multipath virtual circuit switched networks each session distributes its traffic among a set of available paths. If all possible paths are considered as available, then the source decision on its traffic distribution can be considered as routing. We model the routing operation as a mathematical problem that distributes the input traffic over possible paths for each session. In this paper we will introduce a distributed and iterative algorithm which will keep the average end-to-end delay for individual paths in the required bound, in addition to minimizing the total average delay of all packets in the network. The convergence of the algorithm will also be shown.**

Index Terms— **traffic Distribution, routing, Convex optimization, Ssubgradient method**

I. Introduction

CComputer networks have evolved into a new generation in which a wide range of new services are provided to various network users[1]. For many of these new services, such as VOIP, IPTV, Network Games, and so on, it is not sufficient to transfer the information to the destination but for the user's satisfaction, it is necessary to guarantee user's required quality of service as well. In this way new services with arbitrary quality of service requirements can be deployed in the network. Providing the quality of service guarantees for services should be achieved using the least possible resources of the network such that the network can be optimized in terms of resource utilization[2].

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Network optimization algorithms determine best routes and traffic distribution over those routes for a given traffic demand for optimum resource utilization. But the research results show that providing quality of service in the cases that routing is performed without paying attention to the QoS requirements is difficult. Therefore, considering the required quality of service in the optimization algorithms and determining the routes accordingly is one of the challenges of the next generation networks and has been the focus of several recent research activities in the field of networks [3].

Recently several new services have become popular in the internet whose quality is dependent on the end to end delay experienced by the packets in the network [4-8]. For an acceptable quality of service (QoS) it is required that the end-to-end delay is kept under a threshold level depending on the type of the service. Providing QoS guarantees is not easy in datagram networks. In new generation networks, such as MPLS, virtual circuit switching is implemented such that the QoS guarantees can be easily implemented.

Most of the QoS-based routing algorithms in the literature provide mechanisms to guarantee the delay for a given route. Nen Jin, et.al. [9], show that to provide QoS in a DiffServ network, the price per unit of traffic rate for each traffic class can be adjusted. They assume a given route for a user and the satisfaction of the user is modeled by a convex function of the traffic amount sent to that route and the QoS of the traffic class.

In [5] QoS is proposed to be provided by adjusting the capacity allocated to each DiffServ class. The QoS measure is the exact proportion of the average delay of two different traffic classes. Each user's traffic is routed through a predetermined path and depending on the amount of traffic of each class, the traffic over the path experiences a delay as its cost.

In [6] a dynamic method is used to adjust the users' traffic rate such that a minimum rate and a maximum delay threshold are guaranteed. A predetermined path for routing the traffic is assumed and the rate is assigned to the users is by solving a convex optimization problem which satisfies the user's delay requirements.

Most of the papers that study traffic distribution in virtual circuit switched networks assume a set of known paths for each source-destination pair. To simplify the problem, a small set of paths is selected from all possible paths beforehand [10,11].

In most of the papers QoS is measured based on m additive factors and the links are modeled by an m-dimensional weight vector $W=(w_1, \ldots w_m)$. W is a positive vector and its elements represent the level of each one of the QoS factors. As a result a path whose elements are under their threshold levels will satisfy the required QoS and can be selected. In this way the QoS-based routing problem is modeled as a multi-constraint (optimal) problem and since these problems are NP-hard, in most cases heuristic methods are used to solve them [3].

In this paper our objective is to introduce a scalable method to distribute the network traffic over all paths in a virtual circuit switched network to minimize the average delay for all packets as the total cost of the network while guaranteeing bounded end-to-end path delay as the users QoS requirement.

The proposed method is based on the analysis of the traffic distribution problem with delay constraints. As a result, the problem is modeled as a constrained convex optimization problem and the routing algorithm is provided based on analytical resolution of this problem.

In section 2 we will introduce an analytical model for distributing traffic over the paths of a session. We will model the traffic distribution as a constrained convex optimization problem. In section 3 we will use Lagrangian dual method to solve this problem and based on that we will propose an algorithm that can be used to realize the proposed method in a data network. In section 4 we will explain how this algorithm can be implemented in real networks. In section 5 we provide simulation results to show that the proposed method converges and that it can effectively achieve its goals. We will conclude the paper in section 6. The analysis of the proposed model is provided in the appendix.

II. TRAFFIC DISTRIBUTION MODEL

A. Review Stage

The common objective of all routing algorithms is to determine appropriate paths to carry the users' traffic from source to destination. All or part of the users' traffic is assigned to selected routes. Therefore, a direct output of a routing algorithm is the amount of traffic allocated to each route. In fact routing can be modeled as a mathematical problem which determines the distribution of traffic destined to a certain destination over the network graph. In different networks and depending on the different requirements in the networks the appropriate routes and the method of finding them can be different.

In this paper source-destination pairs are assumed to be known and are shown with the set W. Each source-destination pair $w \in W$ is considered as a session and its average traffic is shown with r_w.

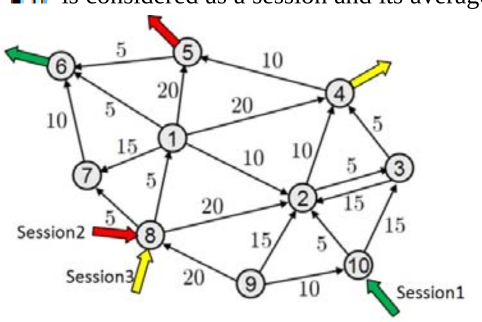

Fig. 1. A network graph with three sessions

We model a data network as a stationary and directed graph G (A, V). Graph nodes that are shown by set V, model the network routers or gateways and graph links, shown by set A, model the physical links between the routers. Some of the nodes of the graph are source and destinations pairs of the sessions in the network (Fig.1)

A session path is a series of graph links that connect the source of the session to its destination. The set of the paths of each session is called P_w. Then, the routing problem will be equivalent to the distribution of each session's traffic over its paths.

Because of limited and different throughputs of the network links, the links are not all the same. We will consider different cost functions $D_{ij}(f_{ij})$ for different paths of each session. In this section we introduce the parameters and notations that we have used in the rest of the paper.

W: The set of all existing sessions, where N_W shows the total number of these sessions.

P : The set of all paths of all sessions $w \in W$ in G

P_w : The set of all paths of session *w*

N_{P_w}: The total number of paths of session *w*.

r_w: Average traffic rate of session *w*.

x_p : Traffic assigned to $p \in P$

th_p: The threshold level of allowed average delay of packets in *p*.

$h_p(x_p)$: The cost function associated with path p

$D_{ij}(f_{ij})$: The cost function associated with link (i,j).

f_{ij}: The flow crossing from link (i,j) of the graph G.

X: A vector of p elements whose pth element is the assigned traffic for path p

Based on the above definitions the following relations hold between them:

$$x_p \ge 0 \quad \forall p \in P \tag{1}$$

$$\sum_{p \in P_w} x_p = r_w \tag{2}$$

$$f_{ij} = \sum_{p|(i,j)\in p} x_p \tag{3}$$

$$h_p = \sum_{(i,j)\in p} D_{ij}(f_{ij}) \tag{4}$$

If the average delay of the packets over a link is considered as the link's cost function $D_{ij}(f_{ij})$ and the messages are delayed only by the links of the network, then (5) expresses the expected delay for all packets over the network [12].Equation (5) indicates the average time that packets remain in the network and consume its resources. Thus it can be considered as the overall system cost.

$$D = \sum_{(i,j)\in A} D_{ij}(f_{ij}) \tag{5}$$

Therefore, even in a virtual circuit network minimizing (5) can be a good objective for traffic distribution since it can improve network resource utilization [13].

In virtual circuit networks, each session's traffic is distributed among the available paths. Assuming a stable network and stationary session traffics, this problem is modeled and analyzed as the problem of distributing the average traffic of each session, r_w, over the set of session's paths, P_w which results in the sessions' path flows, x_p for all sessions. Thus, f_{ij}, the total flow of link (i,j), should be expressed by path flows and , $h_p(x_p)$, the cost of forwarding the flow through each path should also be expressed as a function of its links' cost function. As a result we consider f_{ij} equal to the sum of all path flows traversing link (i,j), (3), and $h_p(x_p)$ equal to the sum of the costs of the links composing the path (4).

In this paper each session represents a customer. The expectation of each customer from the network is defined based on the customer's traffic's delay tolerance. In this case the customer will be satisfied if the average delay is bounded to a certain threshold. Therefore, considering the delay of each link as its cost seems to be appropriate. In this model sum of the cost function of the links which compose the path, is considered as the path cost (4).

Considering (5) as overall cost function of the network and (4) as the delay bound required by the customers, (QoS Requirement), the routing in the network can be modeled as Problem-1:

$$minimize D = \sum_{ij \in A} D_{ij}\left(\sum_{p \mid ij \in p} x_p\right) = \sum_{ij \in A} D_{ij}(f_{ij}) \quad (6)$$

$$\sum_{p \in P_w} x_p = r_w \forall w \in W \quad (7)$$

$$x_p \geq 0 \ \forall p \in P \quad (8)$$

$$h_p(x_p) \leq th_p \forall p \in P \quad (9)$$

In this problem, path flows, x_p, are the variables of the problem. The objective function D is considered as the overall system cost. As a result the purpose of this problem is to find the distribution of the traffic among the available paths to minimize the overall system cost while the constraints (7) to (9) are satisfied. Constraints (7) and (8) guarantee the acceptable allocation of the traffic over the session's paths, and constraint (9) guarantees the delay limitation or users' expectation.

If constraint (9) is ignored, problems-1 is converted to problem-2. Problem-2 is known as minimum delay routing and was first introduced in [13] and then improved in [14-17].

Problem-2:

$$minimize D = \sum_{ij \in A} D_{ij}\left(\sum_{p \mid ij \in p} x_p\right) = \sum_{ij \in A} D_{ij}(f_{ij}) \quad (10)$$

$$\sum_{p \in P_w} x_p = r_w \forall w \in W \quad (11)$$

$$x_p \geq 0 \ \forall p \in P \quad (12)$$

III. Solving the Problem

Usually the cost function $D_{ij}(f_{ij})$ is expressed as a convex, non-decreasing, continuous and differentiable function. As a result, the paths cost will have these characteristics. Since the cost functions $h_p(x_p)$ are convex, problem-1 is a constrained convex optimization problem [18,19], which can be solved using any one of the existing methods to solve such problems, such as Projected Gradient, Interior Point, and so on. But our objective is a solution that can also be implemented in real networks. In this regard we formulate and solve its Lagrang dual problem. In other words since Problem-1 is a convex optimization problem we will use the duality theorem to solve problem-2.In preposition-1 we show that strong duality holds. Since there is practical solution to solve problem-2 [20], the dual problem is described using the Lagrange multipliers related to (9).

Thus the Lagrangian is (13) where we relax only constraint (13) by introducing Lagrange multiplier $\lambda_{p,w}$ for each path $p \in P_w$ and the Problem-3 is the partial dual function [18]

$$L(X, \Lambda) = D(X) + \sum_{p \in P_w} \lambda_p . (H_p(x_p) - th_p) \quad \forall \Lambda \geq 0 \quad (13)$$

Problem-3:

$$q(\Lambda) = minimize_X L(X, \Lambda) \quad (14)$$

$$\sum_{p \in P_w} x_p = r_w \quad (15)$$

$$x_p \geq 0 \ \forall p \in P_w \quad (16)$$

Considering problem -3 as the dual function of problem-1, the dual problem will be Problem-4

Problem-4:

$$maximize \ q(\Lambda) \quad (18)$$

$$\lambda_p \geq 0 \ \forall p \in P_w \quad (19)$$

As mentioned in proposition-2 of Appendix, $-q(\Lambda)$ is a convex function which in general case is not differentiable but it is sub-differentiable at all points. Therefore, problem-4 can be solved iteratively using subgradient method [21]. In this method an initial value is given to variable Λ, (Λ^0), and in each iteration according to (20) a new value is calculated which will be closer to the optimum value.

$$[\Lambda^{k+1} = \Lambda^k + \alpha^k . g^k]^+ \quad (20)$$

To calculate the new value of Λ in k^{th} iteration, first a subgradient, $(-g^k)$, of function $-q(\Lambda)$ is calculated at Λ^k, and then Λ^{k+1} is calculated using (20) in which, α^k is a positive step size and '+' denotes projection on the set $\mathbb{R}^+$.

As result-3 illustrates, for finding a vector g^k we must distribute the traffic base on Problem-3 solution according to $\Lambda = \Lambda^k$. In this case the deviation of the cost of a path, is equal to the associated coordinate of $g^k$the related coordinate is zero.

As a result, the iterative algorithm finding the, Λ^*, best solution for problem-4. Obviously in this way the input traffic have been distribute such as paths flow are solution of problem-3 according to amount of $\Lambda = \Lambda^*$. Since the conditions for strong duality exists according to proposion-1 of Appendix, this distribution will be the optimum solution of problem-1 as well. In the following we will explain the proposed algorithm and will provide its convergence proof in the appendix (Fig.2).

Algorithm Steps:

Step 1: A feasible value is given to Λ. Since in problem 4 every $\Lambda \geq 0$ is acceptable, we use $\Lambda^0 = 0$ as the initial value. In this step, the initial value of q^{best} is 0.

Step 2: In iteration k, problem-3 must be solved based on the value of Λ^k, and the optimum value of $q(\Lambda^k)$ and the optimum point $X^*(k)$ are found. The elements of this vector are shown by $x_p^*(k)$. In other words we must use a mechanism to determine path flows, for a minimum delay routing when (21) is assumed as the cost function of each link. Therefore we can interpret Lagrang multipliers as bottleneck indicators of paths.

$$D_{ij}^k = \left(1 + \sum_{p \mid ij \in p} \lambda_p^k\right) . D_{ij} \quad (21)$$

Step 3: In iteration k and based on the value of $X^*(k)$, calculated in step 2, the amount of deviation of the cost of each path from the threshold level of that path is calculated. Considering the proposition-3, the negative of this value can be considered as the p^{th} element of the subgradient vector $-q(\Lambda^k)$ or $-g_p^k$. After calculating the deviation for all paths, the value of Δ for next iteration or Λ^{k+1} can be calculated using (20).

Step 4: The value of $q^{best} = \min\{q^{best}, q(\Lambda^k)\}$ is calculated and k is increased by one. Then if the conditions of finishing the algorithm is met the algorithm terminates or otherwise it goes back to step 2 for next iteration.

Stopping Condition: Condition of finishing the algorithm: In a simple case, the condition to finish the algorithm can be a maximum number of iterations.

IV. Helpful Hints

As mentioned before, the main objective of this paper is to distribute the traffic of a session over its known paths. A session, as defined in this paper, can be equivalent to a source and destination pair in virtual circuit switching networks such as ATM and MPLS, or in general in any network that uses explicit routing or source routing. Even a certain DiffServ class in these networks can be considered as a session. In practice the algorithm is implemented for each session iteratively and in parallel for all sessions. We assume each iteration of the algorithm is performed in a time slot.

At the end of each time slot k, destination nods calculate the deviation of the average delay for each path from the required delay bound. Then the bottleneck multiplier of the path is calculated based on the deviation and is sent to the source nod. The average delay of packets in each iteration can be obtained by the destination either using analytical modeling or just by measurement. In the case that the path delay is estimated using measurement methods, based on the assumptions about the link cost in this paper, the algorithm will definitely converge according to the proposition-4. During each time slot source nods distribute input traffic according to optimal point of problem-3. In each iteration, the problem-3 is a minimum delay routing problem in which the cost function of each link is defined by (21). This problem can be solved by one the existing methods [13-17].

Each time slot can be in the order of the end to end trip time in the network. The algorithm is scalable because it is implemented for each session separately. Thus, the set of

paths for each session can be assumed to include all possible paths for the session based on the topology of the network, and because of that, the algorithm will practically select the routes. Therefore a separate method for determining the possible routes will not be necessary

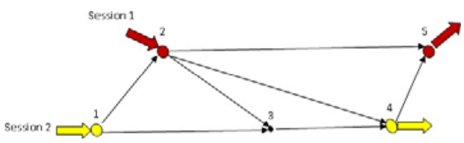

Fig. 1. Algorithm flowchart

V. Simulation

We simulate the algorithm for two sessions over the network graph in Fig.3. The algorithm is independently executed for each session iteratively and synchronously. All possible paths for session 1 are P1, P2 and P3 and for session 2 are P4, P5 and P6 (22a-f).

$P1=\{(1,2),(2,4)\}$ (22-a) $P2=\{(1,2), (2,3), (3,4)\}$ (22-b)

$P3=\{(1,3), (3,4)\}$ (22-c) $P4=\{(2,3),(3,4),(4,5)\}$ (22-d)

$P5=\{(2,4), (4,5)\}$ (22-e) $P6=\{(2,6)\}$ (22-f)

In this simulation the average delay of the paths is modeled as (23) which is a convex, continuous, and differentiable function of its average traffic. In this equation C is the capacity of the path and K is a positive coefficient. The domain of this function covers only traffics between 0 and C and as the traffic gets closer to C the delay increases exponentially. The function is undefined for values above C.

$$D_{ij}(f_{ij}) = \frac{K_{ij} * f_{ij}^2}{C_{ij} - f_{ij}} \qquad (23)$$

The coefficient and capacity of links are shown in Tab1.

Tab1. Parameter of Network links

Link(i,j)	K(i,j)	C(i,j)
(1,2)	2	44.7
(1,3)	4	16
(2,3)	3	44.7
(2,4)	1	44.7
(2,5)	8	44.7
(3,4)	4	44.7
(4,5)	2	16

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We assume constant input traffic that can be considered as the expected value of the sessions' traffic in general. The average input traffic for each session is assumed to be 20 Mbps.

In this simulation we aim to clarify two important points. First, we show that the iterative algorithm converges to optimal point of problem-1. Second the algorithm achieves its goal in limiting the end to end delay of the paths in addition to minimizing the total network delay. Since the main objective of the routing problem is similar to minimum delay routing problem, we compare the results of our algorithm with minimum routing algorithm for the above scenario.

In first step, the path flows for each session are calculated based on the minimum delay routing. Thus we calculate path flows by solving problem-2 using CVX package in MATLAB. In this case we calculate end to end delay for every paths and the expected delay of packets. These results are shown in Tab.2 .

Tab2. Simulation result of step1

link	Flew	Cost
(1,2)	17.92	47.96
(1,3)	2.08	4.98
(2,3)	0	0
(2,4)	25.47	33.73
(3,4)	2.08	6.50
(2,5)	12.45	76.91
(4,5)	7.55	26.97
$\sum D_{ij}(f_{ij})$		197.05

a) Links' Information

Path	Paths flow	Marginal Cost	en2e delay
1	17.92	11.55	81.68
2	0	13.55	54.46
3	2.08	11.55	11.48
4	0	16.74	33.48
5	7.55	14.74	60.7
6	12.45	14.744	76.91

b) Paths' Information

In second step, the path flows for each session are calculated based on the minimum delay routing with end to end delay constraint. The end to end delay bound for each path is supposed to be 76 units in this simulation. Thus we calculate the path flows by solving perоblem-1 using CVX package in MATLAB. The results are shown in Tab. 3.

Tab3. Simulation result of step 2

link	Flew	Cost
(1,2)	17.39	44.28
(1,3)	2.61	8.16
(2,3)	0	0
(2,4)	25	31.72
(3,4)	2.61	10.38
(2,5)	12.39	76
(4,5)	7.61	27.62
$\sum D_{ij}(f_{ij})$	198.16	

a) Links' Information

P ath	Paths flow	Marginal Cost	Lag Multiplier	e2e delay
1	17.39	10.87	0.385	76
2	0	14.91	0	54.66
3	2.61	15.04	0	18.53
4	0	18.74	0	38
5	7.61	14.7	0	59.35
6	12.39	14.62	0.115	76

b) Paths' Information

The total cost of the network in step 2 is close to but higher than the optimum total cost in step 1. Yet in step 1 the individual path cost, for paths 1 and 6, is beyond the end to end delay bound. This means that the algorithm has been able to limit the delay for the price of some increase in the total cost. Also it can be seen that based on the Complementary Slackness conditions, the path flow for paths 1 and 6 is decreased from the optimum value of step 1, down to a point that their average delay is deacreased to the threshold level. As such, the optimum dual variable, DV, of these two paths are expected to be higher than 0 and that of the other paths expected to be 0. It can be interpreted that the marginal cost of the paths 1 and 6 should be lower compared to that of the path 3 for the calculated traffic.

In final step, the suggested algorithm is simulated using MATLAB. The step size is equal to .008. The simulation finishes after 1000 iterations. The final results of the algorithm are shown in Tab. 4. We also show the stepwise results of the algorithm for Lagrange multipliers and two of the link flows as a sample in Fig.4 and Fig.5.

Tab.4. Final result of Phase3 for iteration number 1000 and step size .008

link	Flew	Cost
(1,2)	17.39	44.28
(1,3)	2.61	8.16
(2,3)	0	0
(2,4)	25	31.72
(3,4)	2.61	10.38
(2,5)	12.39	76
(4,5)	7.61	27.62
$\sum D_{ij}(f_{ij})$	198.16	

a) Links' Information

p ath	Paths flow	Marginal Cost	Lag. Multiplier	e2e delay
1	17.39	10.87	0.385	76
2	0	14.91	0	54.66
3	2.61	15.04	0	18.53
4	0	18.74	0	38
5	7.61	14.7	0	59.35
6	12.39	14.62	0.115	76

b) Paths' Information

The results in Tab. 4 are the same as the results in Tab. 3. This means that the iterative algorithm converges to the same result of the central solution.

Fig.4 shows that the path flows converge to the same results as the results of centrally solving problem-1. Fig.5 shows that the Lagrangian multiplires of the distributed solution converge to the optimal dual variable values obtained from centrally solving the problem-1

VI. Conclusion

In this paper a new method for traffic distribution in virtual circuit switched networks and a methodology to implement it were introduced. In this method the input traffic of each session is distributed among the possible paths, such that the total system cost will be minimized at the same time that the average cost for each path is kept bounded below a required threshold level.
This method is scalable as it is per session operation. It is analytically proven in the paper that this algorithm converges under the assumptions that are feasible in real networks. The simulation results approved the effectiveness of the algorithm. The results obtained from the simulation were in line with the results from analytical resolution of the convex optimization problem.

Appendix

In this section the analysis of the proposed algorithm will be provided. First we define some parameters that we will use in this section.

$\bar{x}_p$:Minimum flow of the path p for which the cost gets equal to the threshold level th_p .

$H(X)$: Session w's path cost vector which has P elements in which p^th element represents the cost of p^th path.

$A(X)$: Deviation vector which has P elements in which p^th element represents the deviation of p^th path.

Λ^*: Optimum solution of problem-4 which is a vector with P elements.

λ_p^*: p^th element of the optimum vector Λ^*, which is the optimum Lagrange multiplier of the p^th path.

$\dot{h}_{P_i}(X)$: Increasing series of derivative function of the elements of vector H at points X.

$p_{P_i}(X)$: The series of the paths based on the increasing series of derivative functions of the elements of vector H at points X.

Proposition-1: The optimum solution of problem-4 is equal to the optimum solution of problem-2 (Strong Duality)

Since problem 2 is a convex optimization problem, if the Slater conditions apply then the strong duality will also apply [18]. According to the Slater conditions, at least one strictly feasible point must exist for problem-2. In other words (A1) must hold:

$$\exists X \mid \sum_{p \in P_w} x_p = r_w \ \& \ h_p(x_p) < th_p \ \forall p \in P \ \& \ x_p \geq 0 \qquad \text{(A1)}$$

Consider the vector $\bar{X}$ whose elements are defined in (A2):

$$\bar{x}_p \triangleq \min\{h_p^{-1}(th_p)\} \qquad \text{(A2)}$$

To prove the strong duality, we assume that the input traffic of the session w is less than the amount given in (A3).

$$r_w < \sum_{p \in P_w} H_p^{-1}(th_p) \qquad \text{(A3)}$$

Taken into account the upper limit of r_w, there will exist a traffic vector $\hat{X}$ with the characteristics given in (A4):

$$\hat{X} : 0 < \hat{x}_p < \bar{x}_p \ \forall p \in P \ \& \ \sum_{p \in P_w} \hat{x}_p = r_w \qquad \text{(A4)}$$

Since the functions $h_p(x_p)$ are nondecreasing, and considering the definition of $\bar{x}_p$, inequality (A5) will hold:

$$h_p(\hat{x}_p) < th_p \ \forall p \in P \qquad \text{(A5)}$$

Therefore, $\hat{X}$ meets the condition A1.

$\blacksquare$

Result-1: Assuming that the input traffic of session w meets (A3), strong duality exists and the optimum solution of problem-4 will be equivalent to the optimum solution of problem-2.

Result 2: Because of strong duality, (A6) should hold for the optimum points of problem-2 and problem- 4.

$$\lambda_p^* \cdot \left(h_p(x_p^*) - th_p\right) = 0 \equiv \begin{cases} h_p(x_p^*) - th_p < 0 \ \Rightarrow \ \lambda_p^* = 0 \\ h_p(x_p^*) - th_p = 0 \ \Rightarrow \ \lambda_p^* \geq 0 \end{cases} \qquad \text{(A6)}$$

In other words, at the optimum point of problem 4, the Lagrange Multiplier of the paths whose cost are lower than the threshold level will be equal to 0, and for the paths that their Lagrange Multipliers are larger than 0 the final traffic amount assigned to them will be such that the cost of these paths will be exactly equal to the threshold level.

Proposition-2: A) The function $-q(\Lambda)$ defined in problem-4 is a convex function of λ and B) It has subgradient at all the points in its domain.

If

$$C \triangleq \left\{(x_1, \dots, x_n) \,\middle|\, \sum_{p \in P_w} x_p = r_w, x_p \geq 0 \ \forall p \in P_w\right\}$$

Then

$$-q(\Lambda) = \text{maximize}_{X \in C} - L(X, \Lambda).$$

A) –q is a convex function:

Defining vector A(X) and function b(X) by (A9) and (A10), $-L(X,\Lambda)$ can be considered as a linear function of Λ for a given value of vector X, as in (A11)

$$A(X) \triangleq Th - H(X) \qquad \text{(A9)}$$

$$b(X) \triangleq \sum_{p \in P_w} h_p(x_p) \qquad \text{(A10)}$$

$$-L(X, \Lambda) = (A(X)^T \cdot \Lambda + b(X)) \qquad \text{(A11)}$$

Taking into account the definition given in 11-z for function L, $-q(\Lambda)$ can be considered as the point-wise Maximum of the family of linear functions at all points Λ according to 12-z. Therefore $-q(\Lambda)$ is a convex function [18, section 3-2-2].

$$-q(\Lambda)|_{\Lambda^1} = \max_X\{(A(X)^T \cdot \Lambda + b(X))|_{\Lambda^1}\} \qquad \text{(A12)}$$

B) Function $-q(\Lambda)$ has subgradient at all pints Λ :

$-q(\Lambda)$ is differentiable at all points Λ that only one X, $X^*(\Lambda)$, maximizes (A12), i.e. at these values of Λ, only one of the functions $(A(X)^T \cdot \Lambda + b(X))$ is larger than the others, therefore at these points, the subgradient of the function is unique and is equal to its gradient which is calculated using (A13).

$$\frac{\partial f}{\partial \lambda} = \nabla(-q(\Lambda)) = A(X^*(\Lambda)) = Th - H(X^*(\Lambda))$$
$$\& \qquad \text{(A13)}$$
$$X^*(\Lambda) = \arg\left(\max_X\{(A(X)^T \cdot \Lambda + b(X))\}\right)$$

But at the points Δ that (A12) has maximum at more than one point and more than one of the functions $(A(X)^T \cdot \Lambda + b(X))$ have the largest value at the same time, the function $-q(\Lambda)$ is not differentiable, but it has subgradient which is calculated using (A14).

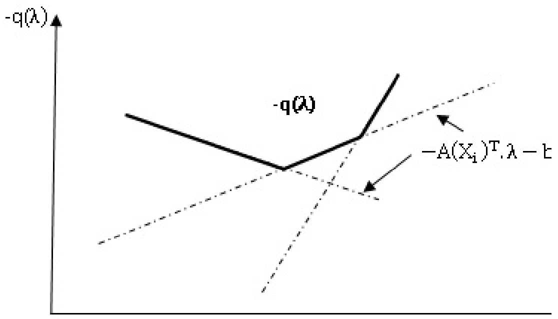

Fig.A1. -q(λ) for one dimensional λ

$$\frac{\partial -q(\bar\Lambda)}{\partial\Lambda}|_{\bar\Lambda} = \text{Convexhull}_{X_i}\{(-A(X_i^*(\Lambda))^T) \mid -A(X_i^*(\Lambda))^T.\Lambda - b(X_i^*(\Lambda))|_{\Lambda} = \}$$
& (A14)
$$X_i^*(\Lambda) \in \arg(\max_X\{(A(X)^T.\Lambda + b(X))\})$$

Therefore, considering (A13) and (A14), subgradient of $-q(\Lambda)$ exists at all points in its domain.

■

According to Proposition-2, the function $-q(\Lambda)$ is the point-wise infimum of a family of affine function (A12), then it is concave and sub-diffrentiable at any points in Proposition-3 an equation is provided to calculate one of the subgradient vectors of function $-q(\Lambda)$ that can be used in the algorithm in Fig.2.

Proposition-3: At each point $\bar\Lambda$ the (A15) gives the subgradient of $-q(\Lambda)$ at that point.

According to (13,14) at given $\bar\Lambda$, given each optimal solution of (12), X_i^*,then $-A(X_i^*)$, is one of the subgradient vectors of $-q(\Lambda)$ at point $\bar\Lambda$. Then according to (A12), the optimal point of (12) at point $\bar\Lambda$, can be obtained from solving problem-3 based on $\bar\Lambda$. Thus by using definition of $A(X)$, A9, we have:

$$-g(\bar\Lambda) = -A(X^*) = (Th - H(X^*)) \in \frac{\partial q}{\partial\Lambda}|_{\bar\Lambda}$$
& (A15)
$$X^* = \arg(\min_{X\in C} L(X,\bar\Lambda))$$

Or X^* is an optimal point of problem-3 based on $\bar\Lambda$.

■

Result-3: Considering (A15) the number of coordinates of vector $g(\bar\Lambda)$ is equal to the total number of paths of session w. The p^{th} coordinate of this vector is equal to the deviation of the cost of path w_p from its threshold level. In this equation the path cost should be calculated when the traffic is the optimum solution of problem-3 for vector $\bar\Lambda$. Therefore to calculate the subgradient vector at point $\bar\Lambda$ it suffices that problem 3 is solved for vector $\bar\Lambda$ such that its optimum traffic vector can be found. Then the cost of each path is calculated for this traffic and its deviation from the threshold level is considered as the element of the subgradient vector

Proposition-4) The algorithm introduced in section 3 converges:

As shown in Fig.2, this algorithm describes the steps of the subgradient method to solve problem-4. According to the proof given in [21], if the value of the subgradient of function $-q(\Lambda)$ in all points has an upper bound such as G and in addition to that the distance from the initial point of the algorithm and the optimum point is less than R, then the subgradient method converges [21]. Therefore, to prove the convergence of the algorithm, first an upper bound or the distance of the initial point of this algorithm and the optimum point is introduced, and then the upper bound for the value of the subgradient vector of function $-q(\Lambda)$ at all acceptable points is calculated.

A) Upper bound for the distance between the initial point Δ^0 and optimum point Λ^*

The initial point of the proposed algorithm in this paper is $\Delta^0 = 0$. The deviation of the cost functions of the paths can be ordered incrementally at points $X^*(\Lambda^0)$ and point $\bar X$ as (16) and (17).

At this point we assume that input traffic of the session hold (A3), such that we can make sure that at least one strictly feasible point exists for problem-2. According to appendix 1the dual problem will have an optimum solution. If Λ^* is the optimum solution of the dual problem (problem-4), and the optimum solution of problem-3 for this Lagrange Multiplier is $X^*(\Lambda^*)$ [20], Considering the necessary and sufficient optimality conditions in this problem, equations (A18) and (A19) will apply [20]:

$$(1+\lambda_p^*).\frac{\partial h_p}{\partial x_p}|_{x_p^*(\lambda_p^*)} = (1+\lambda_{\hat p}^*).\frac{\partial h_{\hat p}}{\partial x_{\hat p}}|_{x_{\hat p}^*(\lambda_{\hat p}^*)}$$
$$\forall x_p^*(\lambda_p^*), x_{\hat p}^*(\lambda_{\hat p}^*) \neq 0 \qquad \text{(A18)}$$

$$(1+\lambda_p^*).\frac{\partial h_p}{\partial x_p}|_{x_p^*(\lambda_p^*)} \leq \frac{\partial h_{\hat p}}{\partial x_{\hat p}}|_{x_{\hat p}^*(\lambda_{\hat p}^*)=0}$$
$$\forall x_p^*(\lambda_p^*) \neq 0, x_{\hat p}^*(\lambda_{\hat p}^*) = 0 \qquad \text{(A19)}$$

If, $\lambda_{p_{PN_{F_w}}}^*$, the Lagrange multiplier associated with the path p_{PN_w}, that has largest derivative of cost function at pint $X^*(\Lambda^*)$ is not zero. According to Result-2 this path's flow is not zero. Therefore, its cost function must be equal to its threshold value. Then according to (A18) the other Lagrange multiplier must also be none zero and the cost function of the other paths are equal to their threshold levels. Since the cost functions are non-decreasing and convex, this result contradicts (A3). Thus $\lambda_{p_{PN_{F_w}}}^*$ is equal to zero and A20-A22 must hold :

$$\lambda_{p_{PN_{F_w}}}^* = 0 \qquad \text{(A20)}$$

$$\lambda_{p_1}^* \leq \frac{\dot h_{p_{N_{F_w}}}(x^*(\Lambda^*))}{\dot h_{p_1}(x^*(\Lambda^*))} - 1 \qquad \text{(A21)}$$

$$\lambda_{p_i}^* \leq \frac{\dot h_{p_{N_{F_w}}}(x^*(\Lambda^*))}{\dot h_{p_i}(x^*(\Lambda^*))} - 1 \leq \lambda_{p_1}^* \qquad \text{(A22)}$$

Considering equalities (A21) and (A22), the largest element of vector Λ^* is its p1 element.
Since functions are non-decreasing convex functions their derivative will be non-decreasing as well and therefore:

$$h'_{p_{N_{P_W}}}\left(X^*(\Lambda^*)\right) \leq h'_{p_{N_{P_W}}}(\bar{X}) \qquad \text{(A23)}$$

$$h'_{p_1}\left(X^*(\Lambda^0)\right) \leq h'_{p_1}(\bar{X}) \qquad \text{(A24)}$$

If the elements of the gradient of vector H in all points $0 \leq X \leq \bar{X}$ is bounded, every one of the elements of vector Λ^* will be smaller than the value calculated in equality (A25).

$$r=\frac{h'_{p_{N_{P_W}}}(\bar{X})}{h'_{p_1}(X^*(\Lambda^0))}-1 \qquad \text{(A25)}$$

Therefore, for all values of input traffics for which the problem has a feasible solution, and assuming that the elements of the gradient vector H are bounded at all points $0 \leq X \leq \bar{X}$, the upper bound of R can be calculated using (A26):

$$R \leq N_{P_W}.r \qquad \text{(A26)}$$

b) Upper bound on the norm of the subgradient vector

Based on the definition of the increasing series of derivatives of the cost functions for all available points X, the equality (A27) holds.

$$h'_{p_i}(X) \leq h'_{p_{N_{P_W}}}(X) \quad \forall p_i \qquad \text{(A27)}$$

Since the cost functions of the paths are non-decreasing and convex functions, their derivatives will also be non-decreasing. Therefore, for all points X (A 28) will hold.

$$\text{If } 0 \leq X \leq \bar{X} \text{ then } h'_{p_{N_{P_W}}}(X) \leq h'_{p_{N_{P_W}}}(\bar{X}) \qquad \text{(A28)}$$

As a result, considering the definition of the subgradient vector in equation (A25), and using equations (A27) and (A28), (A29) can determine the upper bound for the value of vector g at all points Λ.

$$\|g(\Lambda)\| \leq G \leq N_{P_W}.h'_{p_{N_{P_W}}}(\bar{X}) \qquad \text{(A29)}$$

Based on the maximum distance between the initial point and the optimal point of the algorithm in (A26), and the upper bound calculated for the subgradient at every step of the algorithm, the subgradient method for solving this problem will converge.

∎

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